A COMPARISON OF THE KINEMATIC GPS AND AEROTRIANGULATION RESULTS COMPLETED FOR THE NEW BRUNSWICK COLOUR SOFTCOPY ORTHOPHOTOMAP DATA BASE PROJECT

A STUDY COMMISSIONED BY WATERMARK INDUSTRIES INC.

BY

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EXECUTIVE SUMMARY

The Geodetic Research Laboratory of the University of New Brunswick was contracted by WaterMark Industries Inc. to evaluate the equivalence of two photogrammetric photo-centre coordinate data sets. More specifically, it was to be determined whether or not a set of GPS-determined coordinates is commensurate with a corresponding set of coordinates determined by way of aerotriangulation (AT).

The methodologies used for the coordinate comparison were: one, to directly compare the corresponding coordinates by means of simple differencing; and two, to transform one set of coordinates to the coordinate system of the second set via a seven parameter transformation and analyse the transformation residuals.

A systematic mean bias of 25.5 m was observed between the labelled geodetic height components of the two data sets. When the AT geodetic heights were, instead, compared with the GPS heights labelled as "orthometric" the mean bias was reduced to 5.2 m. This implied that either the AT or GPS geodetic heights were incorrectly labelled and we therefore took the "orthometric" GPS heights as "geodetic" for our subsequent analyses. With this substitution, there exists systematic mean differences of 0.8 m, -0.65 m, and 5.2 m in latitude, longitude, and height, respectively between the two data sets. These biases cannot be completely explained by the datum difference between the two sets of coordinates, given that the expected biases are approximately only 0.2 m, 0.2 m, and 0.5 m for latitude, longitude, and height, respectively.

The results of the transformation show, as expected, that the component biases have been removed. The remaining r.m.s. noise or deviations of the transformed GPS coordinates from the AT coordinates is between approximately 2.0 m and 2.5 m for each Cartesian component.

The level of significance of the post-adjustment noise levels is a function of the accuracy of the given AT and GPS coordinates. Four scenarios were tested, based on given accuracy levels of the data sets. The results indicate that assuming a 3.0 m AT and

a 0.4 m GPS component precision at the 68% confidence level, there is no significant difference (at the 95% significance level) between the coordinates of the two data sets after the transformation. If however, the data set precisions are better (i.e., more accurate), then there exists significant differences between the data sets after the transformation.

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1. INTRODUCTION

The analysis described in this report was performed to assess the equivalence of kinematic GPS-derived photo-centre coordinates and those obtained with conventional aerotriangulation (AT) for the *New Brunswick Colour Softcopy Orthophotomap Data Base Project*. The report is a component of a broader feasibility study, investigating of the use of GPS-determined photo-centre positions for control purposes in aerial photography. The comparison was carried out under terms of a subcontract between the Geodetic Research Laboratory of the University of New Brunswick (UNB) and WaterMark Industries Inc.

1.1. CONTRACT OBJECTIVE

The prime objective of the contract was to transform the coordinates from the coordinate frame of one of the data sets to the other, and determine whether or not significant differences exist between the final two sets of coordinates. This would indicate if GPS-derived photo-centre positions could be used in place of positions determined by way of traditional AT. More generally, the objective was to determine if the accuracy of the GPS-determined photo-centre coordinates was equivalent to that of the AT-determined coordinates.

1.2. DATA SETS

The data sets considered consist of processed AT and GPS measurements from Block 1 of the *New Brunswick Colour Softcopy Orthophotomap Data Base Project*. The area of interest approximately encompasses the region from 46.4°N to 47.1°N and from 64.6°W to 65.6°W. The AT curvilinear coordinates were provided by GeoNet Technologies Inc. in the NAD83 (CSRS) datum. The GPS-derived photo-centre curvilinear coordinates were provided by Airborne Sensing Corporation in the NAD83 (1989) datum.

1.3. REPORT OUTLINE

In the second section, the methodologies used for comparison are described, along with the rationale for the particular tools used. Section three contains the results of the comparisons and analysis of the results. This section also identifies the possible GPS error sources in photogrammetry, and addresses the crucial issue of the significance of the AT and GPS-derived coordinate discrepancies. Conclusions are reported in section four. In section five, recommendations for future work are given including procedures for the use of kinematic GPS for photo-centre determination and for combined GPS/photogrammetry. Finally, references are given in section six.

2. METHODOLOGIES USED FOR COMPARISON

Given that the data set coordinates refer to different datums, a transformation is required to compare corresponding coordinate values. According to Service New Brunswick [LeBlanc, 1998], planimetric differences between NAD83(1989) and NAD83(CSRS) are expected to be at approximately the 0.3 m level along the coast, and geodetic height differences of approximately 0.5 m are expected. However, local distortions could produce larger discrepancies, especially in the height component.

Therefore after initial inspection of the two data sets, the main analysis consisted of performing a seven parameter transformation to transform one set of coordinates into the coordinate system of the second data set for comparison. In order to accomplish this task two preliminary tasks had to be completed: a suitable software program had to be enlisted; and the Excel format files containing the data sets had to be filtered and formatted for input to the transformation program.

2.1. DATA PREPROCESSING

A Fortran program was written to filter the coordinates from the two data sets. Criteria included matching common photo-centre identification numbers in both data sets, and removing any photo-centre estimate pairs containing gross errors in coordinate information. During this process, it was observed that rather large differences exist between the AT geodetic heights and the GPS-derived geodetic heights. Smaller discrepancies were observed when the AT geodetic heights were compared with the GPS-derived *orthometric* heights. Figure 1 and Table 1 illustrate the height differences in terms of time series and summary statistics, respectively. There appears to be an approximately 25 m systematic component to the difference between the AT geodetic heights and GPS geodetic heights and a smaller 5 m systematic component between the AT geodetic heights and the GPS orthometric heights. Comparison noise (i.e., standard deviations) approaching 1.5 m are present in both comparisons.

These differences can be explained by one or a combination of the following: (1) incorrect transformation between geodetic heights and orthometric heights for either the AT or the GPS data; (2) incorrect height information usage in either the AT or the GPS data processing; (3) incorrect labelling of the given AT or GPS heights; and (4) local distortions between the two datums. The likelihood of each of these actions being the cause of the problem is the subject of the next paragraph.

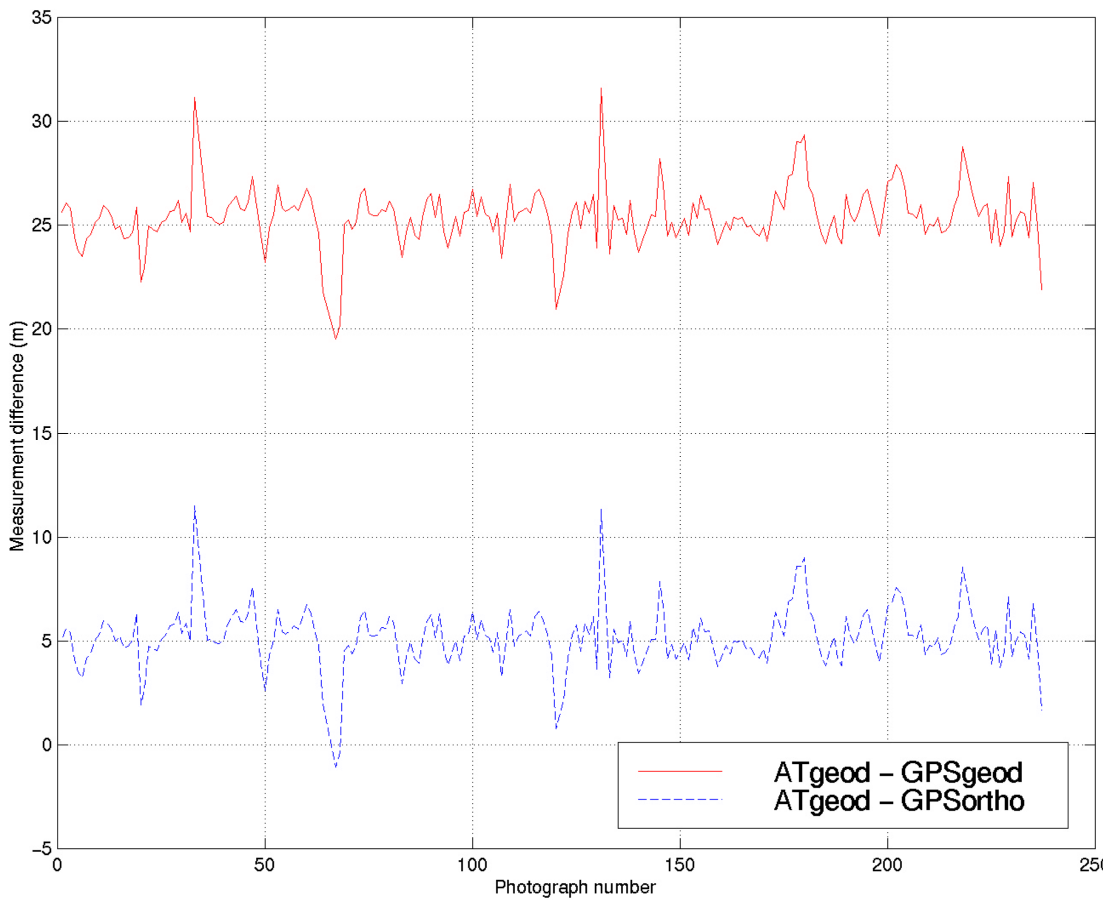

Figure 1: Difference between AT geodetic heights and GPS geodetic and orthometric heights as originally labelled in supplied data sets.

Parameter	$AT_{geod.}$–$GPS_{geod.}$ (m)	$AT_{geod.}$–$GPS_{ortho.}$ (m)
Mean	25.429	5.184
Std. dev.	1.390	1.416
Max.	31.585	11.517
Min.	19.516	-1.078
Range	12.069	12.595

Table 1: Statistics for differences between AT geodetic heights and GPS geodetic and orthometric heights as originally labelled in supplied data sets.

(1) The derivation of the AT geodetic heights can not be verified from the given data set. The transformation between GPS geodetic and orthometric heights were investigated with the use of the Geodetic Survey of Canada *GPS Height Transformation Package* [GSC, 1998]. The geoid (geodetic − orthometric) heights for a subset of data were tested, and it appears that the applied transformations in the given GPS data set are correct. (2) The use of incorrect height information also can not be investigated with the given data sets. (3) Improperly labelled height information, AT "geodetic" being "orthometric" or GPS "orthometric" being "geodetic", could account for the bulk of the systematic component, but a 5 m difference would still exist. (4) Finally, as has been stated, the expected shifts between heights in the two datums is 0.5 m, which would explain only a portion of the height difference. Local distortions would increase the discrepancy, but it is unlikely that the height differences would reach the 5 m level.

Based on the above analysis, it was presumed that, for this report comparison, there exists a labelling error in the GPS data set and "orthometric height" means "geodetic height", and that the remaining height difference is due to datum shifts and artefacts of the production of either, or both of, the AT and GPS coordinates. That is, it is possible that a systematic error of up to 5 m exists in the heights of one set of coordinates.

2.2. DIRECT COORDINATE COMPARISON

The most straightforward method by which to compare and contrast the two data sets is simply differencing the corresponding coordinate values and examining the differences. Even though the coordinates are in slightly different coordinate systems, such a direct coordinate comparison would reveal any systematic as well as random coordinate differences. Matlab scripts were written to compute the differences and produce graphs of the results and associated summary statistics. The results are presented in section 3.1.

2.3. TRANSFORMATION PARAMETER ESTIMATION

As was stated at the outset of section 2, it was decided that a seven parameter transformation was to be used to transform the coordinates of one system to the other for coordinate comparison. By analysing the photo-centre residuals after the adjustment, the degree of misfit of the measurements as compared to the mathematical and statistical model can be gauged. That is, the discrepancies between the two sets of coordinates with the difference due to coordinate systems removed, can be quantified. The Geolab software package [Geolab, 1993] was initially chosen to accomplish this task, however a number of problems were encountered. The relatively small differences of the AT and GPS Cartesian geocentric coordinates caused numerical instabilities in processing. To remove this problem, a local origin was used for one system, which of course produces large translation estimates. Another problem introduced by these data sets was that all of the coordinates are in a localised area and produces poor geometry for estimating the seven parameters (three translations, three rotations, and one scale factor). Again, the use of a local origin helps to overcome this obstacle. Finally, it was discovered that the coordinates in one system had to be held fixed for the adjustment to be performed using Geolab.

It was decided, given the above problems, to develop in-house software to solve the estimation problem, and to use Geolab to test restrictive adjustment scenarios to verify the UNB software's performance. A Matlab program was written using the Molodensky-Badekas algorithm (see, e.g., Harvey [1986]), which properly accounts for the described problems and hence produces correct transformation parameter estimates for the given data sets. The transformation model is represented by

$$\begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix} = \begin{bmatrix} xc \\ yc \\ zc \end{bmatrix} + \begin{bmatrix} tx \\ ty \\ tz \end{bmatrix} + sR \begin{bmatrix} x_A - xc \\ y_A - yc \\ z_A - zc \end{bmatrix}$$

where coordinates in the $(x, y, z)_A$ system are transformed to the $(x, y, z)_B$ system; (xc, yc, zc) are the centroid coordinates for the $(x, y, z)_A$ system; the translation terms (tx, ty, tz) are the coordinates of the origin of the $(x, y, z)_A$ system in the frame of the $(x, y, z)_B$ system; s is the scale factor; and R is an orthogonal rotation matrix represented by the following given small rotations

$$R = \begin{bmatrix} 1 & \kappa & -\theta \\ -\kappa & 1 & \omega \\ \theta & -\omega & 1 \end{bmatrix}$$

where ω, θ, and κ are the rotation angles about the x, y, and z axes, respectively. Subsequent Matlab scripts were written to produce graphs and associated summary statistics of the estimation results and coordinate comparisons.

3. RESULTS AND ANALYSIS

In this section the results of the straightforward coordinate differencing and the transformation estimation are presented in the form of graphs and statistical summaries. The data are represented in terms of geocentric Cartesian coordinates (i.e., x, y, z) and where directly possible, curvilinear coordinates (i.e., latitude, longitude, geodetic height). The graphs (Figures 2 through 10) include time series plots, most of which couple all aircraft flight lines in sequence and choropleth maps, representing a fitted surface based on the data values, giving a more illustrative, but interpolated, sense of the coordinate differences.

Given the discrepancies in the coordinates observed and the assumed confidence that can be placed in the AT coordinates from their past use, the possible GPS error sources in photogrammetry are explored to increase understanding and to possibly explain the produced results.

Finally, using *a priori* knowledge pertaining to the input data sets, the significance of the differences in the data set coordinate values are analysed, with the aim being to reach the final goal of stating whether or not the GPS-derived photo-centre coordinates have an accuracy equivalent to that of the AT-determined coordinates.

3.1. DIRECT COORDINATE COMPARISON

To initiate the investigation, straightforward coordinate differencing was applied to assess the randomness of the discrepancies between the data set coordinates. Figure 2 illustrates the curvilinear coordinate component measurement differences for the amalgamated time series. It must be noted that one arcsecond of latitude is equivalent to about 21 metres and one arcsecond of longitude is equivalent to about 31 metres in the survey region. Summary statistics are given in Table 2. As can be seen, there is a random constituent in the differences with some correlation of the variations between the components. The mean differences (biases) are 0.80 m and -0.65 m for the latitude and longitude, respectively. The 5.18 m systematic height difference was previously observed.

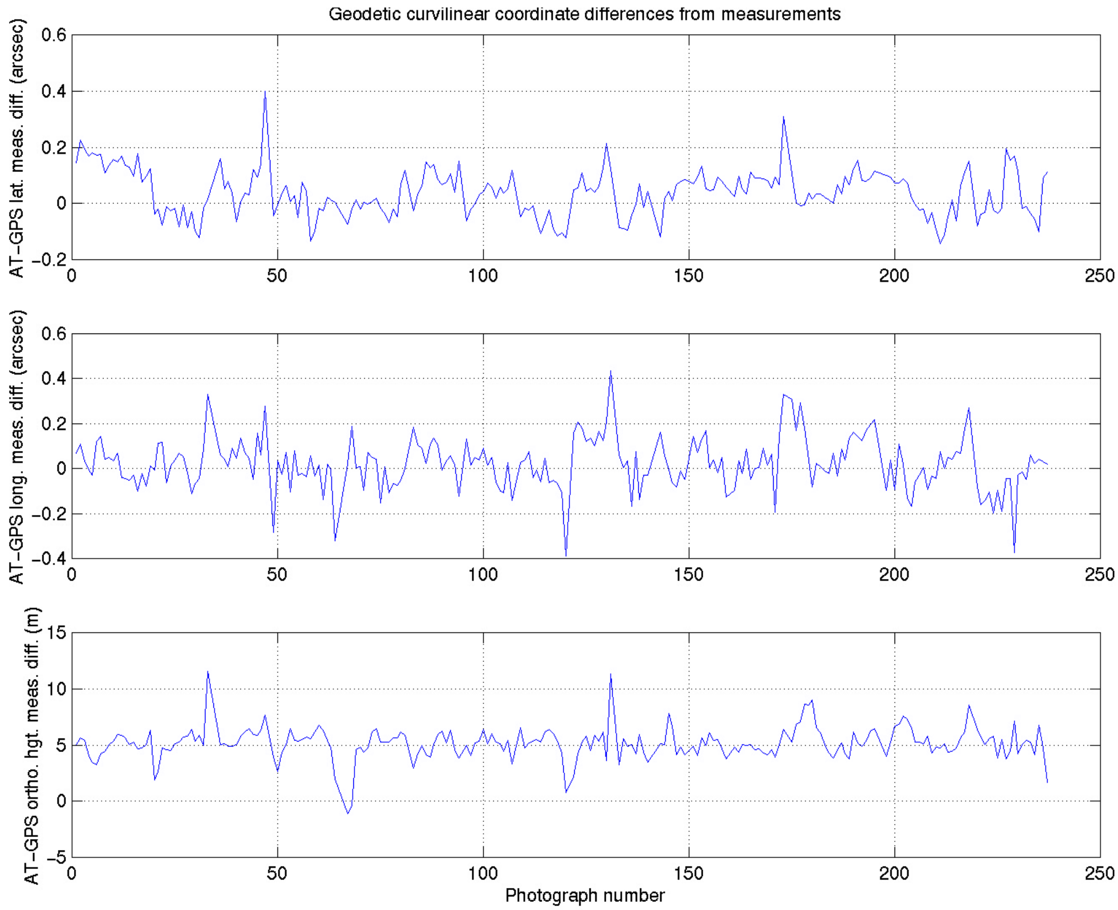

Figure 2: Curvilinear coordinate component measurement differences.

Statistic	Lat. diff. (arcsec)	Long. diff (arcsec)	Geodetic hgt. diff. (m)
Mean	0.038	-0.021	5.184
Std. dev.	0.084	0.114	1.416
Max.	0.398	0.434	11.517
Min.	-0.143	-0.389	-1.078
Range	0.541	0.823	12.595

Table 2: Summary statistics for curvilinear coordinate component measurement differences.

To observe if there are systematic errors occurring on individual flight lines, the curvilinear component differences are plotted flight line by flight line in Figures 3, 4, and 5. As can be seen, there are no apparent systematic effects that are functions of the flight lines.

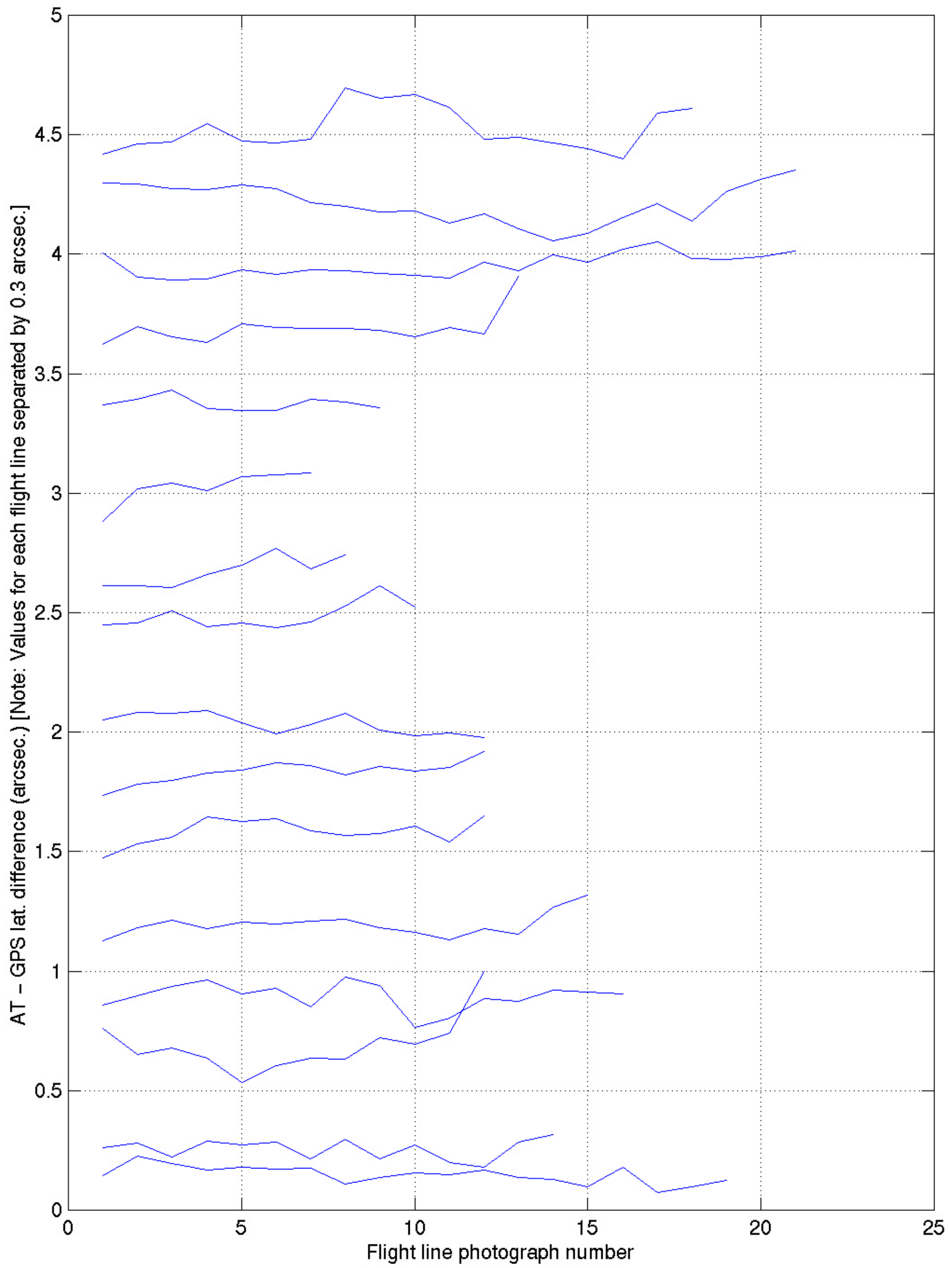

Figure 3: AT – GPS latitude measurement differences for each flight line.

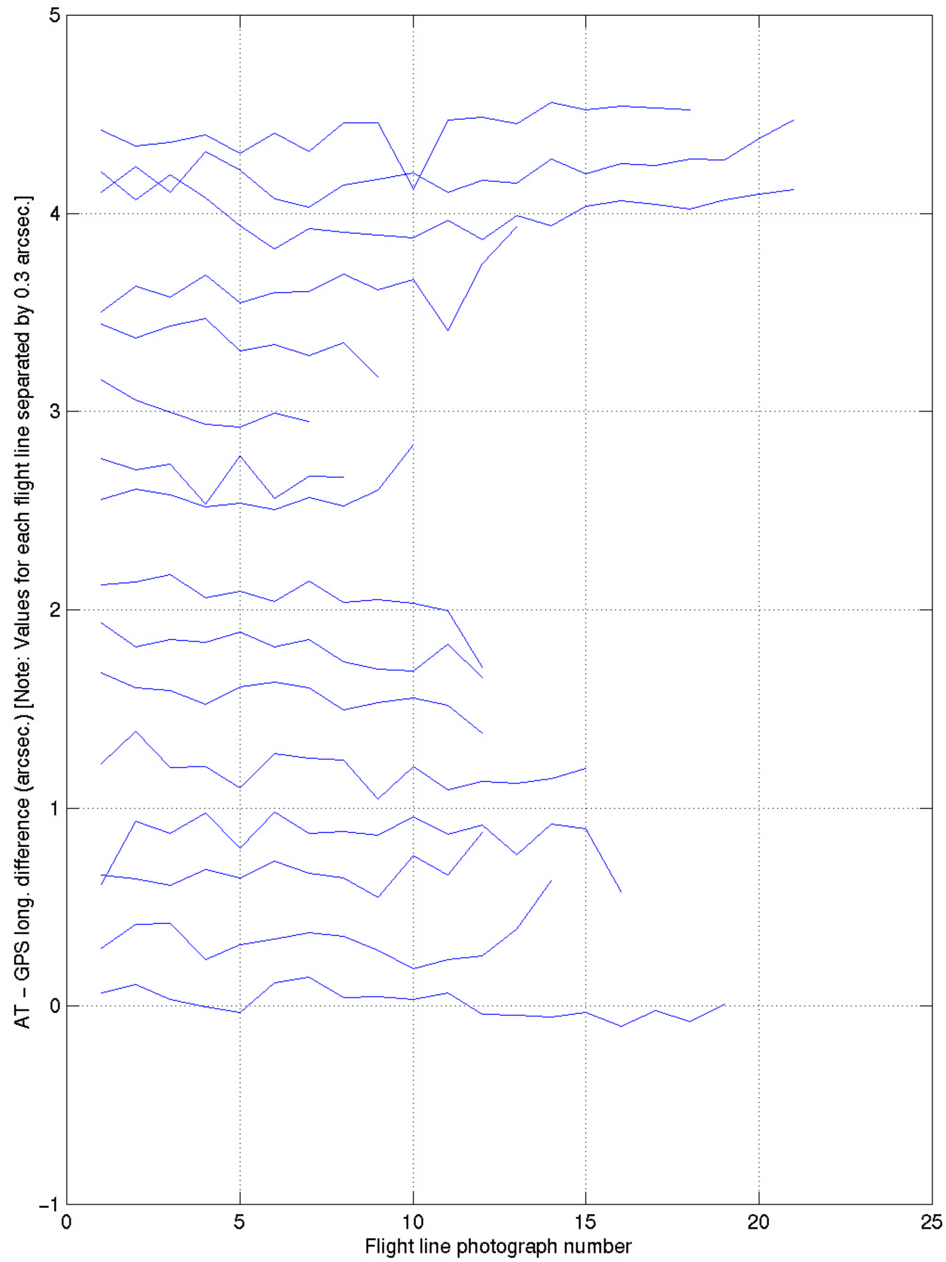

Figure 4: AT – GPS longitude measurement differences for each flight line.

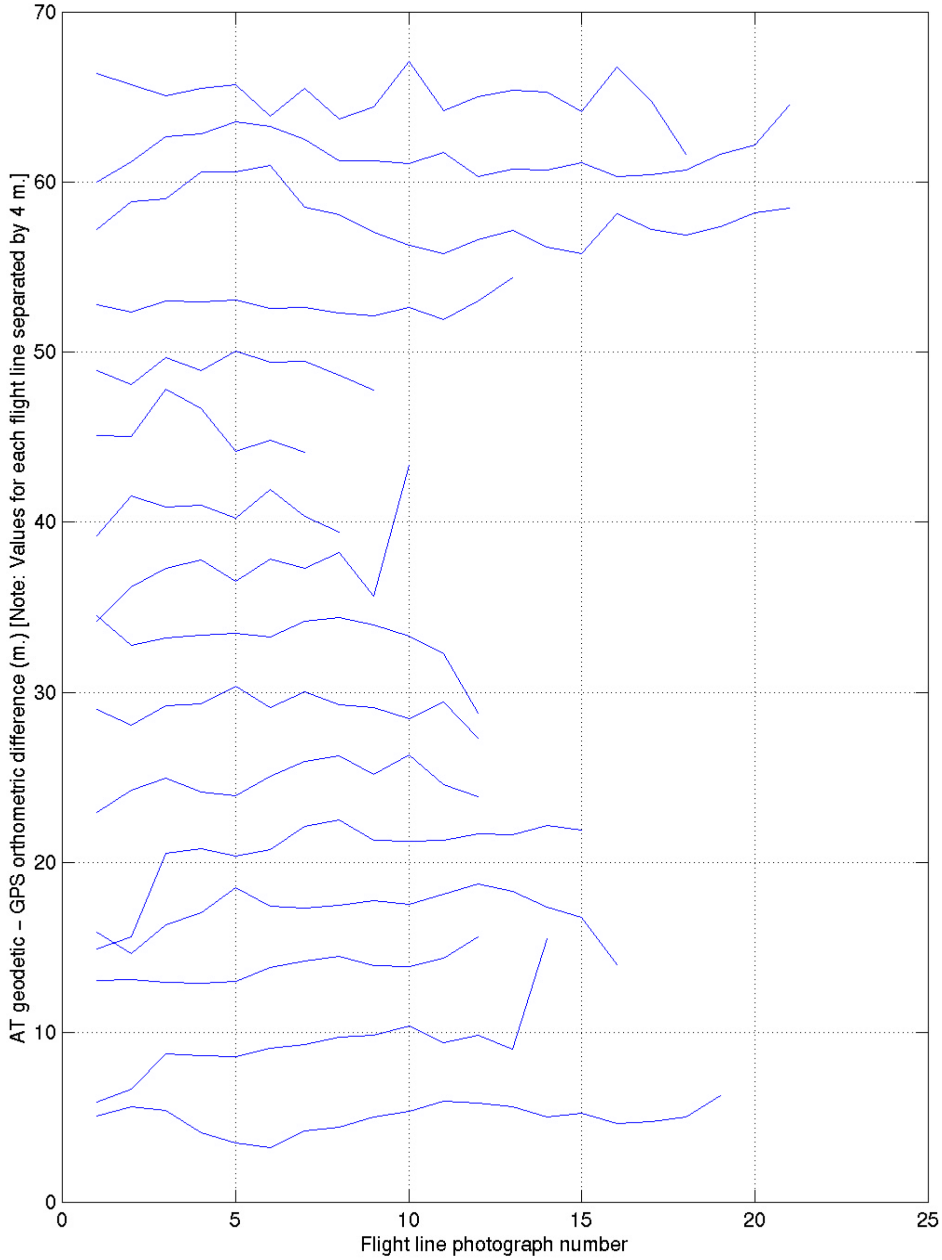

Figure 5: AT geodetic – GPS orthometric measurement differences for each flight line.

The line graphs presented are suboptimal for comparing discrepancies in adjacent flight lines. Therefore in an attempt to glean more understanding from the discrepancies, via a more spatially representative view, choropleth maps have been prepared. Using developed Matlab scripts, a uniform surface grid was interpolated from the discrepancy results using Delaunay triangulation-based cubic interpolation. Caution must be used when interpreting these maps, given that much interpolation has been performed and noting that the survey area was not such a simple diamond shape, but rather a complicated polygon for which values were extrapolated for grid cells along the eastern and western boundaries.

Figure 6 provides a surface representation of the curvilinear component measurement differences. Noting that the aircraft flew along parallels, the discrepancies appear to be spatially random. A discrepancy peak at approximately 46.8°N, 64.8°W appears also to be random given that, due to lack of data in the region, some values were extrapolated beyond the flight lines.

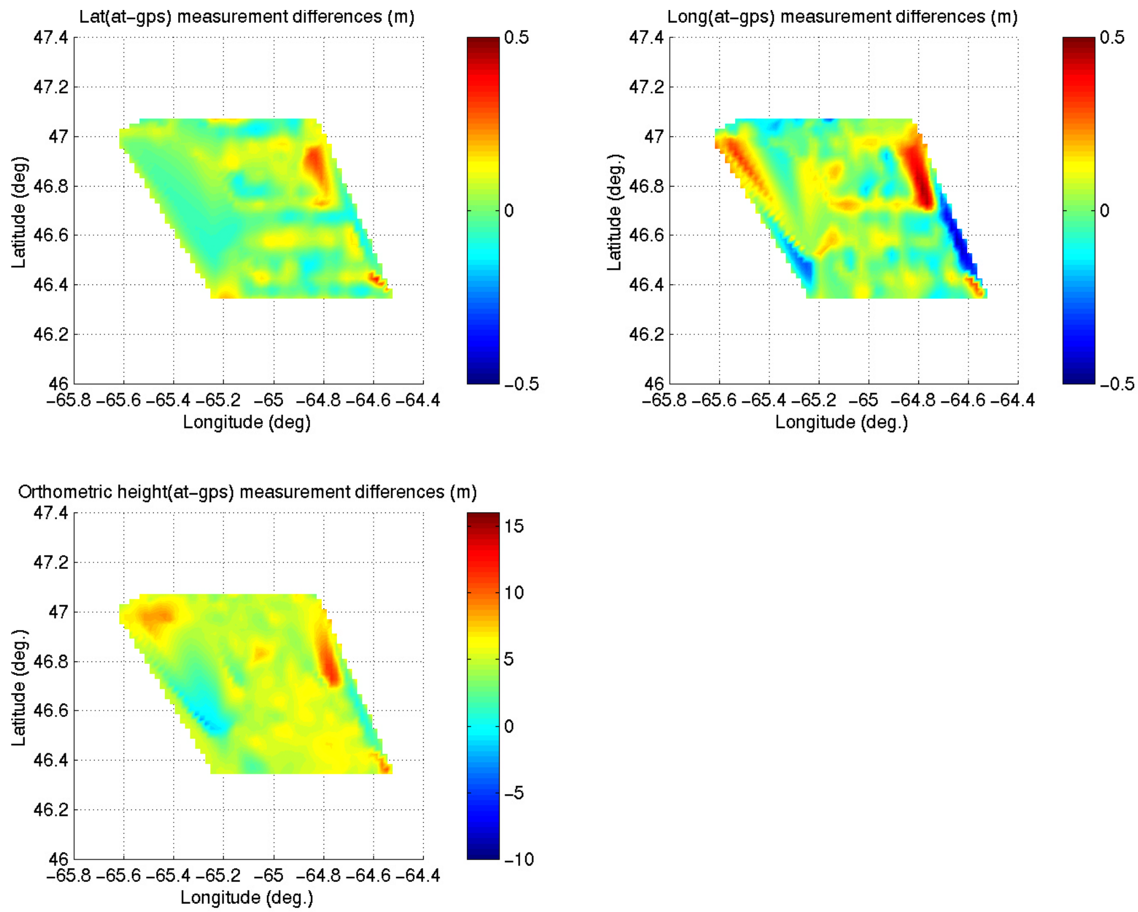

Figure 6: Choropleth maps of curvilinear coordinate component measurement differences.

Figures 7 and 8 and Table 3 provide parallel information to Figures 2 and 6 and Table 2, except that the former describe the Cartesian discrepancies between the data sets.

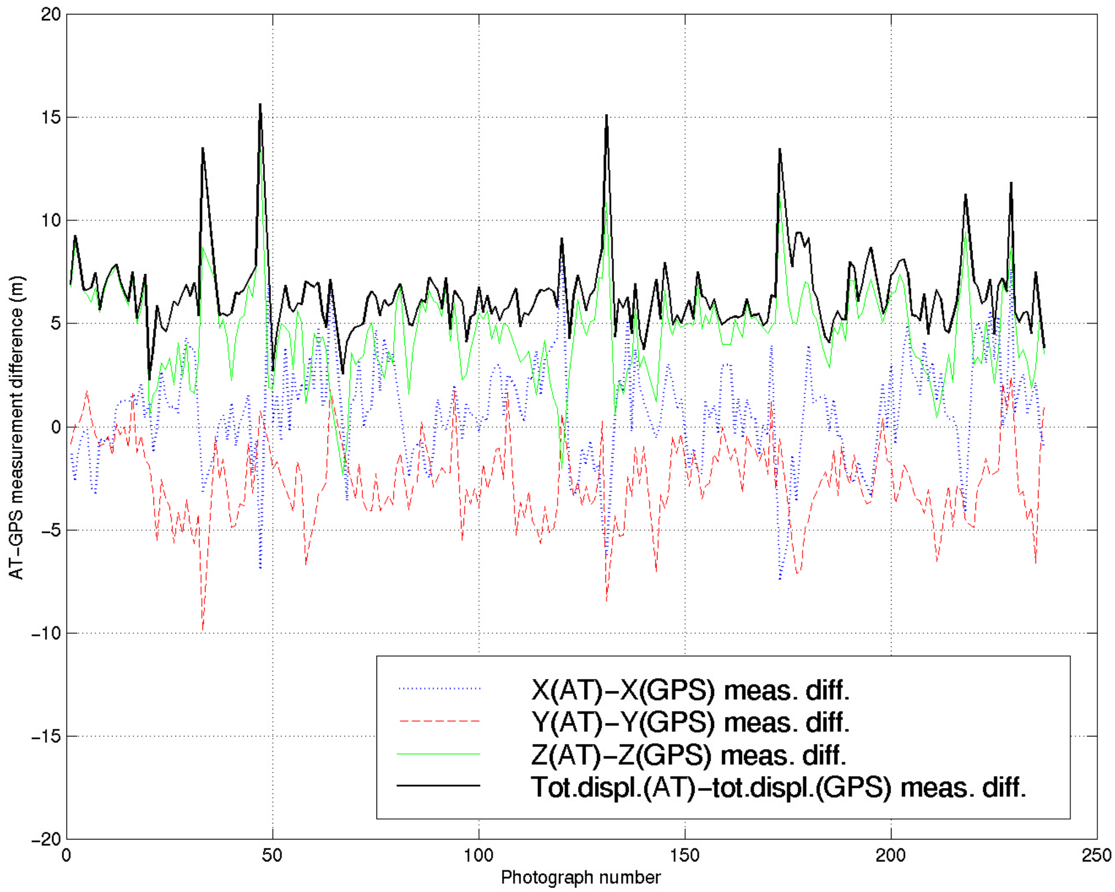

Figure 7: Cartesian coordinate component measurement differences.

Statistic	X (m)	Y (m)	Z (m)	3drss (m)
Mean	1.979	2.821	4.623	6.320
Std. dev.	2.469	1.972	2.131	1.726
Max.	8.888	2.383	14.005	15.654
Min.	-7.473	-9.885	-2.339	2.268
Range	16.361	12.267	16.344	13.386

Table 3: Summary statistics for Cartesian coordinate component measurement differences.

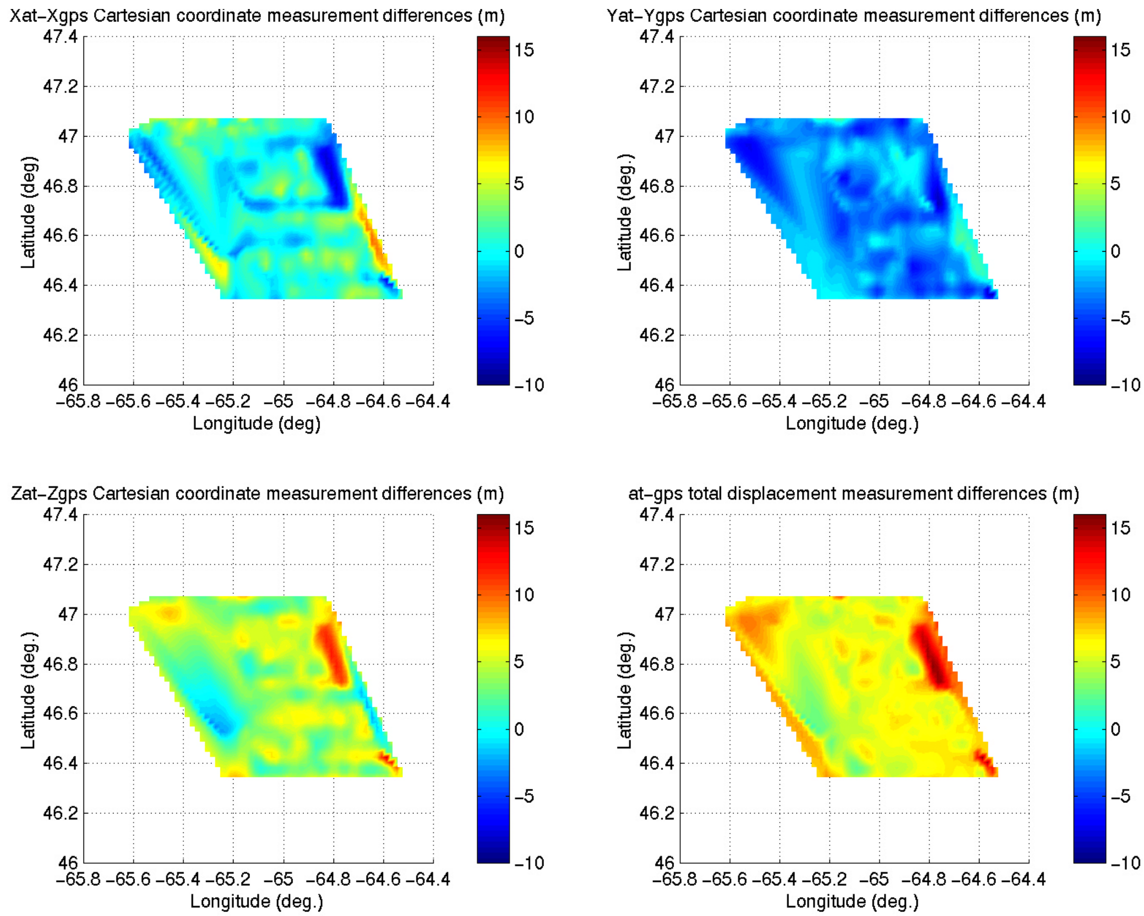

Figure 8: Choropleth maps of Cartesian coordinate component measurement differences.

In summary, aside from the 5.2 m systematic height discrepancy, smaller systematic effects can be observed in latitude and longitude components of 0.8 m and –0.65 m, respectively. The discrepancies in terms of each flight line and on a spatial bases appear random.

3.2. TRANSFORMATION PARAMETER ESTIMATION

The initial estimation strategy involved holding the AT coordinates fixed, considering them to have been produced by the traditional, within specification, survey technique. It was therefore possible to compare the results from the UNB-produced software and the commercial Geolab package. The resulting comparable parameter estimates were essentially the same. The UNB transformation parameter estimates and associated standard deviations are given in Table 4. Note that aside from the three dimensional translation, the rotations and scale factor are inconsequential (statistically insignificant) considering that their associated uncertainties are of the same order of magnitude as their parameter values. At the centroid of the survey area, the three translations can be approximated as -0.80 m in latitude, +0.65 m in longitude, and -5.18 m in height.

Parameter	Estimate	Standard deviation
X translation (m)	-0.736	0.149
Y translation (m)	2.638	0.149
Z translation (m)	-4.577	0.149
X rotation (arcsec)	1.967	1.661
Y rotation (arcsec)	-0.907	1.287
Z rotation (arcsec)	-0.756	1.572
Scale factor (ppm)	4.480	4.469

Table 4: Results of seven parameter transformation estimation holding AT coordinates fixed.

Given that the AT coordinates are assumed correct in the sense of the required specifications, the GPS coordinate residuals from the transformation indicate coordinate differences between the two data sets. These Cartesian coordinate residuals are plotted in Figure 9 and associated summary statistics are given in Table 5. As would be expected from the adjustment process, the coordinate residuals have a zero mean. That is, the transformation removes the bias component of the discrepancies between the two data sets, as can be seen by the equivalence of the negative translation estimates with the mean curvilinear coordinate measurement differences from section 3.1. The standard deviations in Table 5 represent the remaining noise and range from approximately 2.0 m to 2.5 m in each Cartesian component.

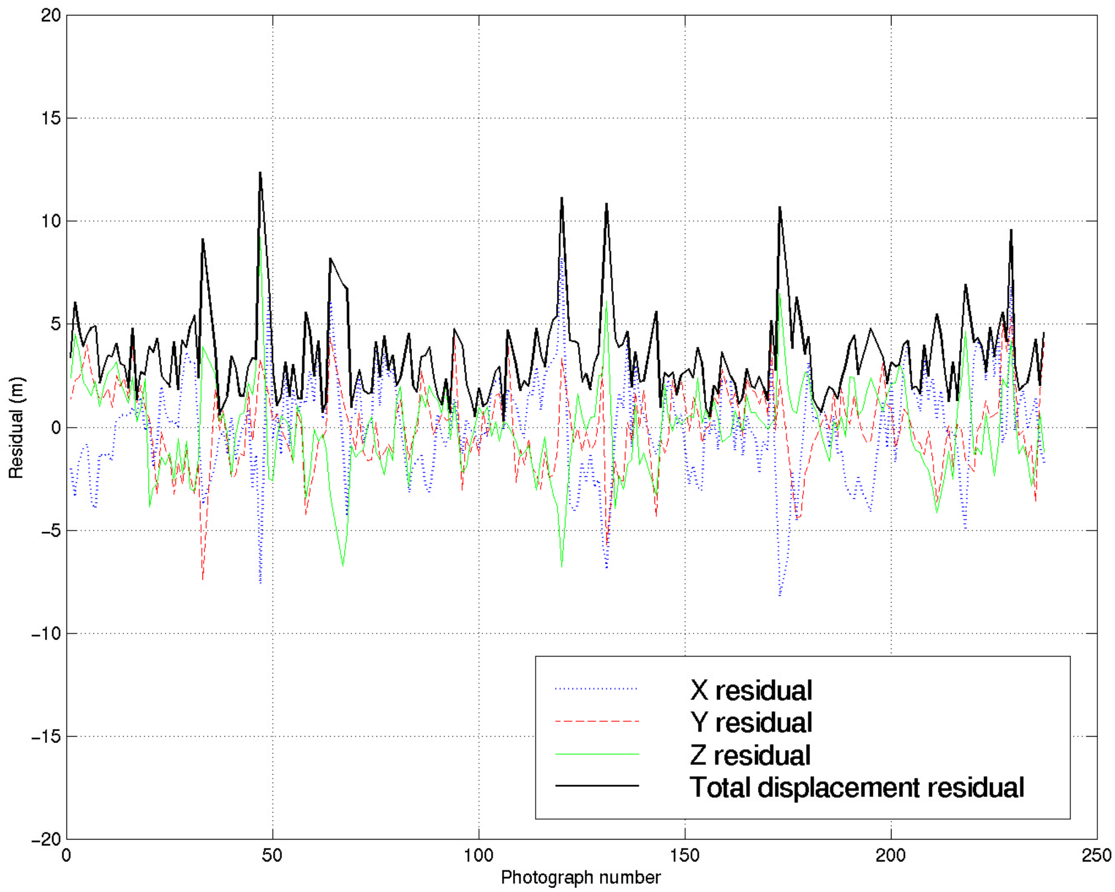

Figure 9: GPS Cartesian coordinate component residuals from seven parameter transformation estimation.

A more spatially representative view of these results is depicted below in the choropleth maps of Figure 10. Again, the results at the east and west sides of the maps are distorted due to lack of data in those areas.

Statistic	X (m)	Y (m)	Z (m)	3drss (m)
Mean	0.000	0.000	0.000	0.000
Std. dev.	2.467	1.968	2.123	1.925
Max.	8.218	5.301	9.226	12.389
Min.	-8.196	-7.371	-6.791	0.313
Range	16.414	12.672	16.017	12.076

Table 5: Summary statistics for GPS Cartesian coordinate component residuals from seven parameter transformation estimation.

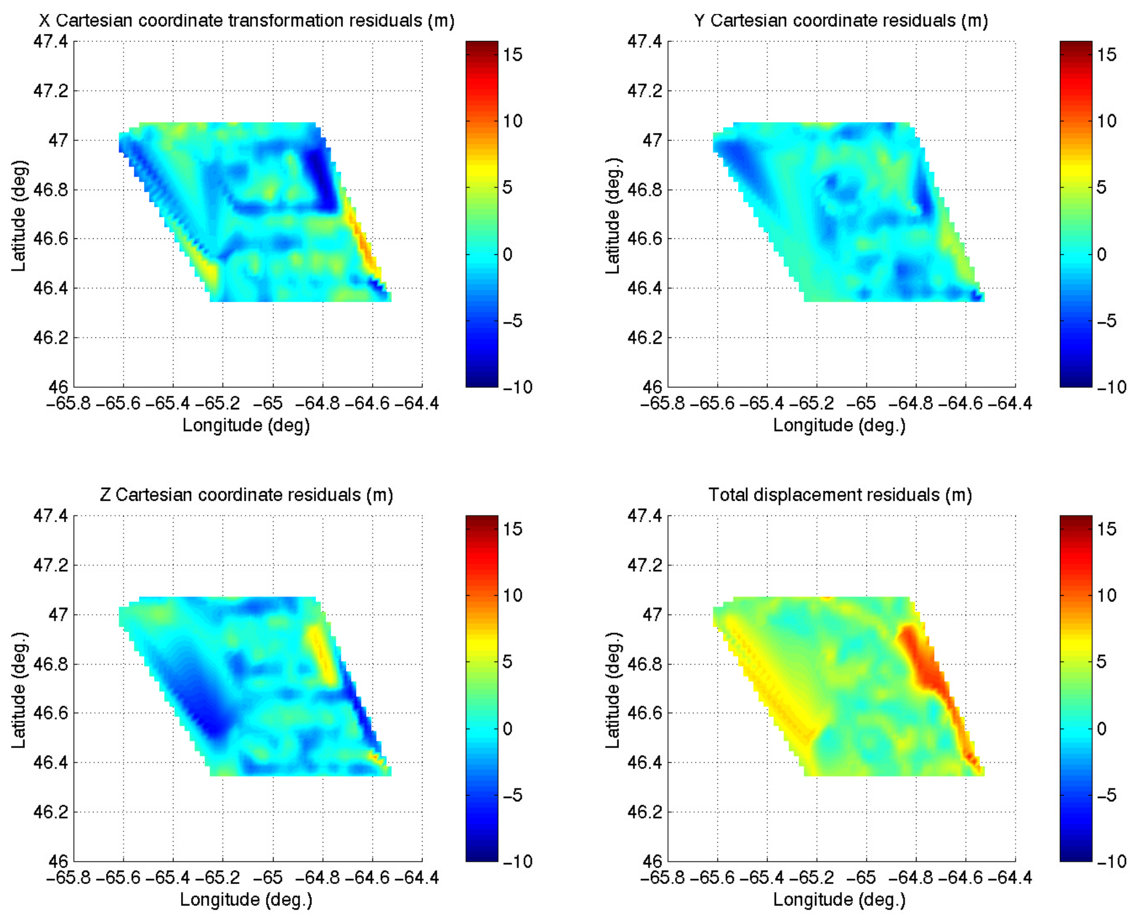

Figure 10: Choropleth maps of Cartesian coordinate component residuals from seven parameter transformation estimation.

In summary, the use of the transformation removes the effects of the systematic latitude, longitude, and height discrepancies. However, these biases are larger than expected for the difference between these two datums as described in section 2. Therefore, this suggests that a portion of the systematic differences could be biases in either of the two data sets and could be as large as: 0.6 m, -0.45 m, and 4.7 m in latitude, longitude, and height, respectively. Also, the remaining discrepancies appear to have no spatial correlation − that is, they appear random and thus are true discrepancies between the data sets that can not be removed by the transformation. The remaining question is how significant are these discrepancies given the precision with which each data set was determined. This is the subject of section 3.4.

3.3. GPS ERRORS SOURCES IN PHOTOGRAMMETRY

Given the discrepancies observed and the assumption that the GPS coordinate values are under scrutiny in this investigation, it would be advisable to briefly examine the sources of GPS error in aerial photogrammetry. Errors can arise in two ways: from the GPS antenna position determination process, or from the GPS-to-camera position transformation.

3.3.1. Kinematic GPS antenna position error sources

Kinematic GPS can provide 95% probability position accuracies from the order of 100 m to 1 cm depending on the hardware used to collect the data and the algorithms used to process the measurements. Any errors in kinematic GPS antenna position estimation will directly impact photo-centre coordinate estimates at the same error level.

For metre- or sub-metre-level positioning accuracies, receivers capable of recording the carrier phase observable must be employed in a differential or relative positioning strategy. That is, data must be collected simultaneously at both the aircraft and at a stationary terrestrial location of known position (a base station). If the aircraft/base station separation is great (e.g., greater than 30 to 40 km), dual frequency receivers must be used to correct for the effect of the ionosphere. In order to receive full benefit from the carrier phase data, the ambiguity terms associated with them must be estimated and fixed to their correct integer values. Proper estimation allows for cm-level positioning results (see, e.g., Mader [1996]). If these ambiguities are not determined correctly, e.g., due to signal interruptions from aircraft manoeuvres, so-called GPS drift errors occur in the position estimates. For example, it has been shown [Ackermann, 1997] that a few carrier phase cycles error would cause few-decimetre, nearly linear errors in the Cartesian components of aircraft antenna position over a half hour interval.

3.3.2. GPS antenna position-to-camera centre position error sources

There are two constituents involved in transforming GPS antenna positions to camera centre positions. Firstly, the antenna position at the time of exposure must be estimated via interpolation since exposures rarely coincide in time with GPS position recordings. Secondly, the interpolated antenna positions must be converted by means of a three-dimensional transformation to camera centre position at the exposure time.

GPS antenna positions are usually recorded at or near a frequency of one hertz. Interpolation tests over 1 s intervals indicate repeatabilities of a few centimetres in each Cartesian component [Lapine, 1996]. It must be noted that GPS-driven shutter activation would remove this need for interpolation. The camera centre position is then determined via a three-dimensional orthogonal transformation that incorporates the spatial offsets between the camera centre and the antenna phase centre and a priori estimates of the elements of the exterior orientation (see, e.g., Curry and Schuckman [1993] or Lapine [1996]). The resulting error depends on the accuracy of the orientation parameters.

3.4. SIGNIFICANCE OF DIFFERENCES BETWEEN THE DATA SETS

The transformation removes the systematic discrepancies in latitude, longitude, and height, irrespective of whether these biases are due to the datum differences or the measurements. In order to interpret the significance of the remaining observed noise differences, the precision of the two data sets must be characterised. That is, by including the information regarding the errors associated with each data set, it can be determined if the observed noise is within the tolerance of the combined errors in the two data sets.

The best metadata for this task are the variance-covariance matrices resulting from the adjustment of each data set. After much investigation, WaterMark was able to provide only approximate accuracies to attach to the input data sets. The AT coordinates have an accuracy of 1-3 m attached to them according to "photogrammetric experts", and the GPS coordinates after re-processing by the GPS contractor varied within 40 cm. With regards to the latter metadata, GPS re-processing should not alter position estimates at all, but this value was applied in the following analysis as it is the only available quality measure.

In order to utilise this information, a number of assumptions had to be made about its meaning. Firstly, the values quoted are assumed to be at the one sigma or the 68% statistical confidence region level. And secondly, it was assumed that the values could represent either total displacement precisions (i.e., 3drss values), or equal component precisions (i.e., in x, y, and z). In terms of the AT accuracy specifications, the 1 m and 3 m total displacement accuracies would translate into equivalent component precisions of 0.6 m and 1.7 m, respectively. And the 0.4 m GPS total displacement accuracy would translate into a 0.2 m component precision. Therefore four input precision scenarios were tested and are represented in Table 6.

Scenario	AT component precision (m)	GPS component precision (m)
1	0.6	0.2
2	1.7	0.2
3	1.0	0.4
4	3.0	0.4

Table 6: Input precision scenarios assumed for variance testing in transformation adjustment.

A rigorous approach to testing the significance of the observed noise levels after the adjustment is to compare the input measurement (*a priori*) variances with the post-adjustment (*a posteriori*) variances via the χ^2-distribution test at a particular significance level, usually 95%. Failure of this test indicates one of three possibilities: problems in the mathematical model; non-normal distribution of the residuals; or incorrect scaling of the measurement variances − that is, the fit of the data to the mathematical model is not consistent with the precision with which the measurements were collected. The model we used is a standard model for such transformation and one would not expect any deficiencies to be associated with it. Analysis shows that the residuals are distributed fairly normally. Therefore the failure of the χ^2 test will indicate that inconsistencies exist between the input measurement precisions and magnitude of the residuals.

The χ^2 test is a confidence interval test. At the 95% significance level, with 650 degrees of freedom, and an *a priori* variance factor of one, the interval is 0.89 to 1.11. The results for the four weighting scenarios described in Table 6 are given in Table 7. As can be seen, all of the tests fail. However failure to the right of the interval indicates that the measurement precisions were too high to produce the given residuals, whereas failure to the left indicates that the measurements precisions were too low to produce the given residuals.

Scenario	*a posteriori* variance factor	pass / fail
1	12.55	fail to right
2	1.59	fail to right
3	4.18	fail to right
4	0.53	fail to left

Table 7: *A posteriori* variance factors determined for four differently weighted transformation adjustments (confidence interval of 0.89 to 1.11).

Therefore if the accuracy of the AT and GPS photo-centre coordinates are 3 m and 0.4 m in each Cartesian component, respectively, the observed post-transformation noise is insignificant and the AT and GPS coordinates are equivalent. If however the two data sets are more precise as outlined in scenarios one through three, there exists significant differences between the two sets of post-transformation coordinates.

4. CONCLUSIONS

The primary goal of this investigation was to determine whether or not there is a significant difference between the given GPS-based photo-centre coordinate data set and the AT-determined photo-centre coordinate data set. Based on our analyses, we offer three main conclusions:

- A systematic bias exists in the height, latitude, and longitude components of either the GPS or the AT data or both.

The straightforward differencing of the geodetic heights from the two data sets show a mean difference of approximately 25.5 m, with a standard deviation of 1.4 m about this mean. It was noticed that the GPS labelled *orthometric* heights were more compatible with the AT geodetic heights and therefore they were used in the analysis, assuming a labelling error in the given data sets. However, a substantial systematic difference in height still exists, with a mean of 5.2 m and again a standard deviation of 1.4 m. Biases of 0.8 m and –0.65 m in the latitude and longitude components, respectively were also observed in the differencing.

Given that the expected biases due to the differences between the NAD83(1989) datum of the AT coordinates and the NAD83(CSRS) datum of the GPS coordinates are 0.5 m, 0.2 m, and 0.2 m for height, latitude, and longitude, respectively, unless very large distortions exist in the survey region, biases exist in the GPS or AT data. These biases could possibly be as large as 4.7 m, 0.6 m, and –0.45 m for the height, latitude, and longitude components, respectively.

- Few metre-level discrepancies exist between corresponding coordinates in the two data sets, even after a seven parameter coordinate transformation is applied to remove existing biases.

The application of an appropriate seven parameter transformation indicates that only the three translation parameters are significant. The transformation removes all of the biases between the data sets, irrespective of whether the bias is due to a datum difference or blunders. The post-adjustment results indicate that the r.m.s. noise level of the coordinate discrepancies are approximately 2.5, 2.0, 2.1, and 2.0 metres for the x, y, z, and total displacement components, respectively.

- Significant differences may exist between the AT and GPS coordinates, depending on the true AT and GPS coordinate noise levels.

The significance of the post-adjustment noise levels are a function of the accuracy of the given AT and GPS coordinates. That is, if the combined uncertainty in the AT and GPS data sets is equal to or greater than the computed after transformation discrepancies, then no significant difference exists between the GPS-based photo-centre coordinates and the corresponding AT photo-centre coordinates. Four scenarios were tested, based on given assumed accuracy levels of the data sets. The results indicate that assuming a 3.0 m AT and a 0.4 m GPS Cartesian component precision at the 68% statistical confidence region level, there is no significant difference at the 95% significance level between the coordinates of the two data sets after the transformation. If however, the data set precisions are better then there exists significant differences between the data sets after the transformation.

5. RECOMMENDATIONS FOR FUTURE WORK

It was assumed by WaterMark that, given the traditional, recognised use of conventional aerotriangulation, any significant discrepancies between the AT coordinates and the GPS coordinates would be attributed to the GPS-derived coordinates. However, given the fact that a third calibration source was not available for this investigation and also the less than compelling conclusions with regards to the observed discrepancies, it was deemed prudent that as a supplementary assignment, recommendations for future GPS/aerotriangulation projects be outlined. This work can be subdivided into the use of kinematic GPS for the determination of photo-centre coordinates for use in aerotriangulation adjustments, or in an integrated approach where certain GPS parameters are estimated in the aerotriangulation estimation process. The following sections contain brief descriptions of these methods with guidelines and empirical results given where possible.

5.1. KINEMATIC GPS FOR PHOTO-CENTRE DETERMINATION

The procedure for kinematic GPS in support for aerotriangulation was basically described in section three where the possible GPS errors were detailed. Specifications for kinematic GPS operations for aerial photography are put forth by the Interdepartmental Committee on Aerial Surveys [ICAS, 1998]. These specifications encompass all requirements for GPS receivers, GPS antennas, the number and location of GPS receivers, offset measurements between GPS antenna phase centre and camera's perspective centre, interpolation of GPS positions to time of exposure, static reference GPS sites, ground control, check points, GPS field logs, GPS solutions, GPS accuracy estimates, and final coordinates. The only item not specifically defined is the GPS processing strategy. It appears that whatever strategy that will provide the appropriate level of aerotriangulation accuracy in terms of photogrammetric model check points is acceptable.

It would be quite a complex task to indicate a specific accuracy attainable with GPS-controlled aerial photogrammetry. Numerous dependent variables are involved in characterising photo-centre accuracy or resultant ground target accuracy. It has been observed for the strategy of using no ground control that sub-decimetre-level accuracy can be achieved for ground targets, which is comparable with conventional photogrammetry (see, e.g., Merchant [1993]). However, this approach is not recommended as complete carrier ambiguity resolution is a prerequisite to attain these results and redundancy is lost without ground control. Gruen et al. [1993] report that sub-decimetre-level Cartesian component GPS minus aerotriangulation GPS antenna coordinate repeatabilites is theoretically attainable using carrier phase data. In other research, Schwarz et al. [1993] propose the use of GPS and INS for camera centre and exterior orientation determination, hence removing the requirement of photogrammetric adjustment to estimate exterior orientation parameters.

5.2. COMBINED GPS/AEROTRIANGULATION

A proven, more successful, method of aerotriangulation with GPS in terms of reliability is an integrated approach. Relative, carrier phase GPS positioning is utilised to provide precise camera centres for the photogrammetric adjustment, but additional GPS-related parameters are also included in the adjustment as well as a number of ground control points. It was observed, as already noted, that improper carrier phase ambiguity selection introduces near-linear drifts in position determinations. These drifts can be estimated in the photogrammetric adjustment. Also, a minimum set of ground control points at the perimeter of the survey area are used in the adjustment for the transformation from the GPS datum to the datum of interest (see, e.g., Ackermann [1997]). It has been shown empirically that the use of four control points at the adjustment block corners in combined GPS/aerotriangulation provides results equivalent to the use of a dense set of control points without airborne GPS data [Gruen et al., 1993].

It is interesting to note that GPS photo centre accuracy is only weakly linked to final adjusted ground coordinates, except for very large scale, high precision scenarios. For one study, as long as GPS photo-centres of less than 20 cm were maintained, final solutions remained similar [Ackermann and Schade, 1993].

In terms of theoretical and empirical accuracy of results as compared with check point positions, averaged root mean square accuracies at approximately the few centimetre-level for horizontal and the decimetre-level for vertical positions were observed for a number of large scale projects in both the theoretical and empirical realm [Ackermann and Schade, 1993]. Finally, to summarise the viability of combined GPS/aerotriangulation, the report *Empirical Results of GPS-Supported Block Triangulation* [Cooper, 1995] was produced, which stated that for small-, medium- and large-scale mapping, the integrated method “safely fulfils the accuracy requirements of aerial triangulation, as the GPS measurements have a precision of the order of 0.10 m or better”.

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