Estimation of the Stochastic Model for Long-Baseline Kinematic GPS Applications

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BIOGRAPHIES

Donghyun Kim is a post-doctoral fellow in the Department of Geodesy and Geomatics Engineering at the University of New Brunswick (UNB), where he has been developing a new on-the-fly (OTF) ambiguity resolution technique for long-baseline kinematic GPS applications and a software for gantry crane auto-steering system using the carrier-phase observations of high data rate GPS receivers. He has a B.Sc., M.S. and Ph.D. in geomatics from Seoul National University. He has been involved in GPS research since 1991 and is a member of the IAG Special Study Group "Wide Area Modeling for Precise Satellite Positioning".

Richard Langley is a professor in the Department of Geodesy and Geomatics Engineering at UNB, where he has been teaching and conducting research since 1981. He has a B.Sc. in applied physics from the University of Waterloo and a Ph.D. in experimental space science from York University, Toronto. Prof. Langley has been active in the development of GPS error models since the early 1980s and is a contributing editor and columnist for GPS World magazine.

ABSTRACT

We propose a new approach for the stochastic model for long-baseline kinematic GPS positioning which can be derived directly from the observation time series under a simple assumption. The performance of our approach was compared with those of existing approaches for the stochastic model – the elevation-angle dependent function approach, the signal-to-noise ratio or alternatively the carrier-to-noise-power-density ratio approach, and the least-squares adaptation approach.

These alternative approaches may have significant limitations in some applications: First, the elevation-angle dependent function is not advisable in kinematic situations because the relationship between antenna gain and the signal elevation angle may be difficult to assess when the antenna orientation is changing which can happen often in kinematic situations. Second, although some GPS receiver manufacturers provide SNR-like values in their data streams, easily-interpreted SNR values are not easy to come by. Third, the least-squares adaptation approach is not advisable for the long-baseline kinematic applications because it is difficult to obtain surplus redundancy in such applications.

Our new approach is free of these difficulties. Although initially developed for long-baseline kinematic applications, it can be used for all situations whether short-baseline or long-baseline, static or kinematic, and for either real-time or post-processing needs.

1. INTRODUCTION

Resolving the GPS carrier-phase ambiguities has been a continuing challenge for sub-centimeter-level high-precision GPS positioning. The GPS carrier-phase ambiguity represents the arbitrary counter setting (an integer value) of the carrier-phase cycle tracking register at the start of observations of a satellite (phase lock), which biases all measurements in an unbroken sequence of that satellite's carrier-phase observations. Once the integer ambiguities are fixed correctly, the carrier-phase observations are conceptually turned into millimeter-level high-precision range measurements and hence it is possible, in principle, to attain sub-centimeter-level positioning solutions. However, resolving the integer ambiguities is a non-trivial problem, especially if we aim at computational efficiency and a high success rate.

To obtain optimal solutions in the least-squares estimation, a functional (or deterministic) and a stochastic model should be specified correctly, where the functional model describes the relationship between observations and unknown parameters while the stochastic model

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represents the noise characteristics of the observations. As has been experienced, the stochastic model is typically more difficult to handle than the functional model when considering a reliable approach for some GPS uses such as long-baseline and kinematic applications.

In general, the stochastic model is involved in three processes of the GPS data processing – the quality control and assurance of the measurements, the ambiguity resolution, and the least-squares estimation. If the stochastic model is not correct, the quality control and assurance process used to detect and fix cycle slips in L1 and L2 carrier-phase observations may not work correctly. The result of faulty cycle-slip fixing can be a disaster in the applications using GPS carrier-phase observations because it introduces artificial biases into the observations and subsequently, the estimated parameter values. An incorrect stochastic model also makes it difficult to resolve correct ambiguities. If the resolved ambiguities are not correct, similar effects to the faulty cycle-slip fixing are transferred to the parameter values. Compared with the effect of the faulty cycle-slip fixing and incorrect ambiguities on least-squares solutions, that of an incorrect stochastic model is less important. However, the quality of the solutions can be interpreted as too optimistic or conservative if an incorrect stochastic model is used.

A preliminary idea of our approach was presented at the ION GPS-2000 meeting [*Kim and Langley*, 2000]. The overall objective of this paper is to justify our approach based on well-defined test scenarios.

2. CORRELATION IN THE OBSERVATION TIME SERIES

When we talk about the stochastic model, we are usually interested in a fully populated variance-covariance matrix. To obtain this matrix, we have to take into account correlation in the observation time series. Three typical physical correlation types exist in the GPS observations: First, cross-correlation which represents the correlation between different observation types (e.g., L1 and L2 carrier-phase, and C1, P1 and P2 pseudorange). Second, temporal correlation which represents the correlation between sequential observations (epoch to epoch). Third, spatial correlation which represents the correlation among 'all-in-view' simultaneous observations. Another possible physical correlation is inter-channel correlation which may exist among the receiver channels. If we use a differencing scheme in the measurement domain, mathematical correlation also exists.

It may be helpful to classify the correlation types in terms of correlation sources. The temporal and spatial correlations are due to biases such as atmospheric effects (i.e., ionosphere and troposphere), satellite orbit bias, multipath and so on. On the other hand, the cross-correlation and inter-channel correlation are due to the receiver signal processing methods.

As we usually experience, correlation makes it difficult to obtain a correct stochastic model. However, correlation is not always so bad because some biases can be canceled due to correlation in the observation time series. For example, spatially correlated biases can be canceled by differencing the observations in the measurement domain. The single- and double-difference are such cases. Applying the same concept, temporally correlated biases can be canceled by differencing the observations in the time domain (e.g., triple-, quadruple- and quintuple-difference). We will take advantage of these concepts in estimating the stochastic model in our approach.

3. PREVIOUS WORK

A brief review in terms of advantages and disadvantages of the previous work on stochastic modeling which has been carried out by many research groups all over the world will be useful in figuring out the problems related to particular GPS applications such as short-baseline or long-baseline situations, static or kinematic mode, and real-time or post-processing operations. There are several approaches which provide somewhat realistic stochastic models: the elevation-angle dependent function approach [*Euler and Goad*, 1991; *Jin*, 1996]; the signal-to-noise ratio (SNR) or alternatively the carrier-to-noise-power-density ratio (C/N_0) approach [*Hartinger and Brunner*, 1998; *Barnes et al.*, 1998; *Collins and Langley*, 1999]; the least-squares adaptation approach [*Han*, 1997; *Wang et al.*, 1998; *Wang*, 1999; *Tiberius and Kenselaar*, 2000]. Fundamental discussions on the observation noise were given by *Langley* [1997] and *Tiberius et al*. [1999].

The elevation-angle dependent function approach takes into account the elevation-angle dependence of the observation noise. This approach is easy to implement in data processing software as long as the parameters of the function are calibrated in the laboratory. However, this approach does not provide information on cross-correlation and spatial correlation. This means that we cannot get the fully populated variance-covariance matrix from the approach. Furthermore, it should be noted that the elevation-angle dependence of the observation noise often varies with the particular kinematic situation. The elevation-angle dependence of the observation noise is induced mainly by the receiver antenna's gain pattern, with other factors such as atmospheric signal attenuation (spacecraft antenna beam-shaping ensures an almost uniform signal field strength independent of elevation angle). The elevation angle is normally computed with respect to the local geodetic horizon plane at the antenna phase center regardless of the actual orientation of the antenna. Accordingly, the relationship between antenna gain and the signal elevation angle may be difficult to assert when the antenna orientation is changing which can happen often in kinematic situations.

The SNR (or alternatively C/N_0) approach provides actual observation noise information which can be derived directly from measurements of the quality of each pseudorange and carrier-phase observation. This information is contained in the SNR measurement. This value determines, in part, how well the receiver's tracking loops can track the signals and hence (to a large degree) how precisely the receiver obtains pseudorange and carrier-phase observations [*Langley*, 1997]. Since multipath signals can adversely impact the receiver SNR depending on whether the direct and reflected signal components reaching the receiver combine constructively or destructively [*Cox et al.*, 1999], this approach does provide realistic observation noise in strong multipath environments. This may be a good approach for the precise point positioning (PPP) technique in which there is not enough redundancy to mitigate the effect of multipath except by down-weighting. Although some GPS receiver manufacturers provide SNR values in their data streams, meaningful SNR values are not always easy to come by (see *Collins and Langley* [1999]). Furthermore, this approach also does not provide the fully populated variance-covariance matrix.

The least-squares adaptation approach, which can be incorporated within a recursive processing scheme such as the Kalman filter and the sequential least-squares estimator, generally provides optimal stochastic models. In addition, it is easy to obtain information for the cross-correlation and spatial correlation from the approach. However, the optimality of the least-squares estimation is not always guaranteed because it depends on assumptions on correlation: e.g., no temporal correlation or a certain order of temporal correlation. The estimates can be too optimistic or conservative if the assumptions are not satisfied. Furthermore, there are particular applications where no observation redundancy is provided. In long-baseline applications, typically we cannot get observation redundancy as long as we take into account all significant biases in the functional model. In such situations, the least-squares adaptation approach will not work.

4. OBSERVATION NOISE ESTIMATION USING DIFFERENCING IN THE TIME DOMAIN

Basically, we try to overcome the three main problems of the existing approaches (i.e., lack of a fully populated variance-covariance matrix, missing temporal correlation, and no observation redundancy in long-baseline applications) in our approach. We assume that double-difference (DD) observation time series are given. Hereafter, we will leave out the notation of DD (or $\nabla\Delta$) for the observations, biases and errors. In short-baseline situations, the effects of the (correlated) biases are usually ignorable. Accordingly, temporal correlation is ignorable in estimating the stochastic model because temporal correlation reflects mainly the behaviour of the biases (although observation smoothing by the receiver will introduce some temporal correlation in observation noise). However, it should be noted that the effect of the double-differenced multipath is not always ignorable even in short-baseline situations. In other words, temporal correlation may exist depending on multipath environments. In long-baseline situations, the biases are not ignorable, so that temporal correlation usually exists in the observation time series.

To remove the non-random behaviour of the observation time series, we use a differencing scheme in the time domain including the triple-difference (TD; differencing consecutive observations after deleting cycle-slip spikes), quadruple-difference (QD; differencing consecutive TD observations), quintuple-difference (dQD; differencing consecutive QD observations), and so on. In this approach, we assume that the effects of any biases can be canceled in the differencing process, so that only the effect of observation noise (assumed as white noise) remains in the resulting time series. This assumption can be justified as long as we can obtain time series with a sufficiently short sampling interval. If the observation time series samples are obtained with a smaller time interval (i.e., a higher data rate) than the time constant of each component of the biases, the assumption can be easily satisfied. This reasoning is based on the fact that differences are generated by subtractive filters. These are high-pass filters damping low frequencies and eliminating constant components. High frequency components are amplified.

There exist two degrees of freedom in our approach: i.e., the order of the differencing and the data rate must be determined in terms of optimality. In general, increasing the order of the differencing continuously is pointless because the time-correlated biases are easily canceled at a low order. The dQD differencing is sufficient for almost all situations according to our analyses. On the other hand, the optimal data rate is usually dependent on particular applications such as static, low-dynamics kinematic, and high-dynamics kinematic applications. Although determining the optimal data rate is more or less arbitrary, there is a general rule which can be understood in terms of the physics inherent in the differencing process: i.e., the data rate should be high enough to make each component of the biases temporally correlated. If the effects of high-frequency (compared with the data rate) components of the biases are significant in the time series, the data rate should be increased to cancel the effects of such components in the differencing process. Most problematic high-frequency component in our approach is the jerk of the geometric range due to moving platform dynamics. Taking a look at the problem in terms of a numerical process, we can get a better insight into how to come up with a solution to the problem. For example, consider the L1 carrier-phase dQD observable:

$$\dddot{\Phi}_1 = \dddot{r} + \dddot{t} + \dddot{s} - \dddot{I} + \dddot{b}_1 + \dddot{n}_1 + \dddot{e}_1 , \tag{1}$$

where Φ_1 stands for the double-difference L1 observable; r for the geometric range; s for the satellite orbit bias; t for the tropospheric delay; I for the L1 ionospheric delay; b_1 for multipath in L1 carrier phase; n for the ambiguities (in distance units); and e_1 for observation noise of the L1 carrier phases. Using the one-dimensional Taylor series including higher-order time derivatives for each of the biases, we have

$$\begin{aligned} S(t) &= S(t_0) + S'(t_0)(t-t_0) + \tfrac{1}{2} S''(t_0)(t-t_0)^2 \\ &\quad + \tfrac{1}{6} S'''(t_0)(t-t_0)^3 + R(t), \end{aligned} \tag{2}$$

where S represents each biases and R is a remainder term known as the Lagrange remainder. Assuming that the observation time interval is $d(=t-t_0)$, we have the following dQD observable:

$$\begin{aligned} \dddot{S}(t_3) &= S(t_3) - 3\cdot S(t_2) + 3\cdot S(t_1) - S(t_0) \\ &= S'''(t_0) d^3 + \Sigma_R(t_3) , \end{aligned} \tag{3}$$

where $\Sigma_R(t_3)$ is the effect of dQD for the remainder R. Substituting Eq. (3) into (1) gives

$$\dddot{\Phi}_1(t_3) = \left[\sum_{\forall S} S'''(t_0) \right] d^3 + \sum_{\forall S} \left[\Sigma_R(t_3) \right] + \dddot{e}_1(t_3) , \tag{4}$$

where

$$\sum_{\forall S} S'''(t_0) = \left[r''' + t''' + s''' - I''' + b_1''' + n_1''' \right](t_0) . \tag{5}$$

Eqs. (4) and (5) clearly show the relationship between the high-frequency components and the data rate in the differencing process. If the effects of the high-frequency components in the right-hand side of Eq. (5) are small enough to be ignorable and/or the data rate (1/δ) is high in Eq. (4), and if the effect of the second term in the right-hand side of Eq. (4) (i.e., the effect of dQD for the remainder R) is also small enough to be ignorable, we can get an acceptable inference as:

$$\dddot{\Phi}_1 \approx \dddot{e}_1 . \tag{6}$$

As long as the differencing process satisfies Eq. (6), the fully populated variance-covariance matrix for the dQD observation noise $\dddot{e}_1$ can be easily estimated using n dQD samples for each double-difference satellite pair time series:

$$\hat{\mathbf{Q}}_{\mathbf{dQD}} = \mathrm{cov}\left[\mathbf{dQD}\right], \tag{7}$$

where $\mathbf{dQD} = \left[dQD_{i,j} \right]$; subscript i stands for a sample number (epoch); subscript j identifies a particular double-difference satellite pair time series number; and $\mathrm{cov}[\cdot]$ is the variance-covariance operator. Since there exists a certain relationship between the original time series and the dQD time series as shown in Eq. (3), assuming that four consecutive L1 observations have the same variance, the fully populated variance-covariance matrix for the observation noise also can be estimated:

$$\hat{\mathbf{Q}}_{\mathbf{DD}} = \frac{1}{20} \hat{\mathbf{Q}}_{\mathbf{dQD}} . \tag{8}$$

Note that the size of samples n should be determined in terms of the unbiasedness of the estimates. In general, the larger the size of samples, the better the estimates. However, there may be a trade-off to some degree in real-time implementation.

5. TEST RESULTS

In order to illustrate the performance of our approach, it has been tested with data sets recorded in static and kinematic modes, and in short-baseline and long-baseline situations. Ashtech Z-XII receivers were used to record dual-frequency data. The summary of the tests is given in Table 1.

Table 1. Summary of the tests

	Test Mode	Baseline Length	Data Rate	Remarks
Test 1	Static	30 m	1 Hz	
Test 2	Kinematic	10 m	1 Hz	Circular motion at low speed
Test 3	Static	80 km	0.1 Hz	
Test 4	Kinematic	80 km	1 Hz	Driving a car at high speed

Firstly, we analyzed the data sets to confirm that the effects of the biases can be canceled in the differencing process. For that purpose, we looked at several particular observation time series: 1) an ideal time series which includes only the observation noise; 2) a time series in which multipath is predominant; and 3) a time series in which the ionospheric delay is predominant.

The ideal time series with weak multipath was obtained from the Test 1 data sets. In static, short-baseline, and weak multipath environments, we can expect that the effects of the biases are ignorable, so that the time series include only the observation noise. But the question is how we can judge observations with weak multipath. A good indicator for multipath is the C/N_0 short-term variability. Figure 1 shows the case of weak multipath.

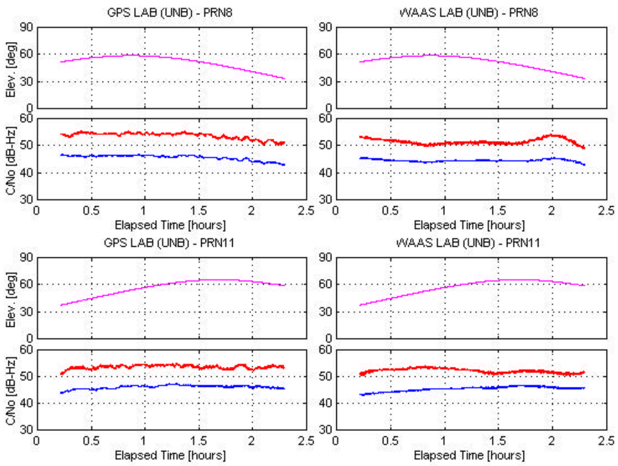

Figure 1. Elevation angle and C/N_0 for a weak multipath situation (Test 1). The red line is the C/N_0 for L1(C1) and the blue line for L2(P2).

To justify the inference for the ideal time series, we estimated the ionospheric delay change using the L1 and L2 TD observations:

$$\begin{aligned} d\hat{I} &= \frac{1}{g-1}\left(TD_{L1} - TD_{L2}\right) \\ &= dI + \frac{1}{g-1}\left(db_1 - db_2\right) + \frac{1}{g-1}\left(de_1 - de_2\right), \end{aligned} \qquad (9)$$

where $g = \left(l_2 / l_1\right)^2 \approx 1.65$ and δ indicates a difference between consecutive epochs. Since the ionospheric delay and multipath are ignorable in the selected situations, the ionospheric delay change estimates $d\hat{I}$ just include the combined effects of the L1 and L2 observation noise. Figure 2 shows justification for that: i.e., there are no significant low-frequency components (e.g., less than 0.1 Hz) which would reflect the effects of the biases. It is more or less arbitrary to define the boundary for the low-frequency components at this moment. However, we will see some reasonable results hereafter. Anyway, we will use the results of this "ideal" time series in comparing with other time series contaminated by biases.

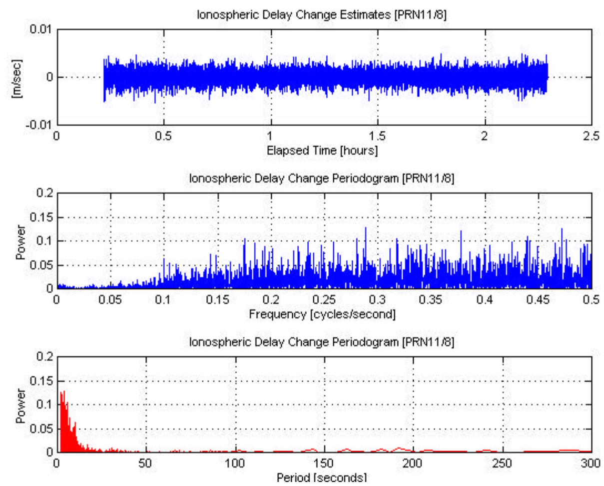

Figure 2. Ionospheric delay change of an ideal time series and its periodogram (Test 1).

To obtain the time series with strong multipath, we selected another time series, which shows a strong multipath pattern as shown in Figure 3 (left side), from the Test 1 data sets. In this case, we can expect that multipath is the only significant bias in the time series.

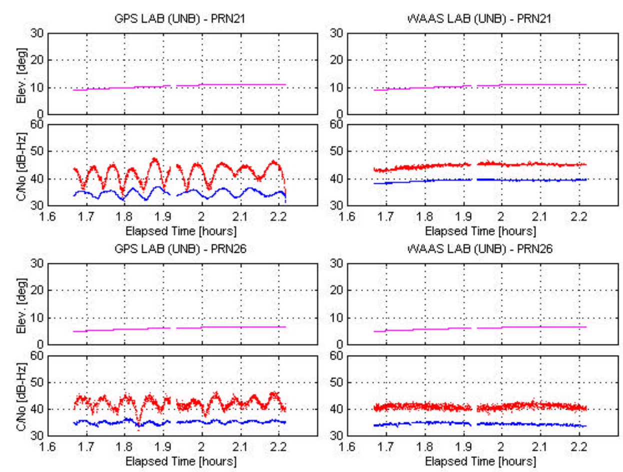

Figure 3. Elevation angle and C/N_0 for a strong multipath situation (Test 1). The red line is the C/N_0 for L1(C1) and the blue line for L2(P2).

Using this particular time series, we can confirm that the differencing process does cancel the multipath. For that purpose, we generated the TD, QD and dQD time series as shown in Figure 4. And then, we looked at each time series in the frequency domain. As the order of the differencing increases, the approximately 3-minute spectral component of multipath disappears as shown in Figure 5. Although our justification for multipath is correct, it is not always the case in kinematic mode. We will highlight that point when we discuss the fully populated variance-covariance matrix estimation later. Our approach allows us to separate, to some degree, the effect of multipath "noise" from other effects contributing to the overall observation noise.

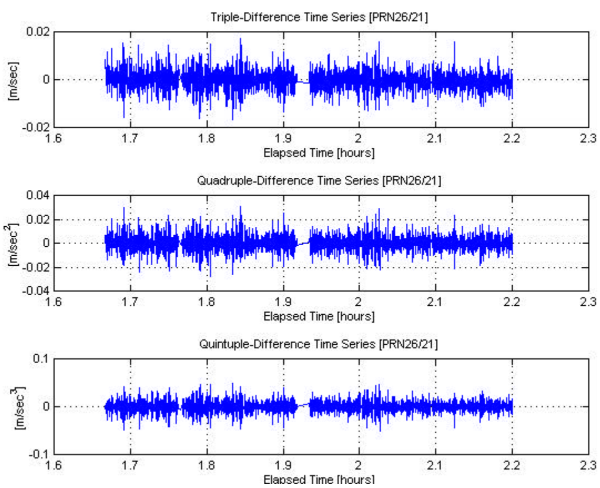

Figure 4. Time series with strong multipath (Test 1).

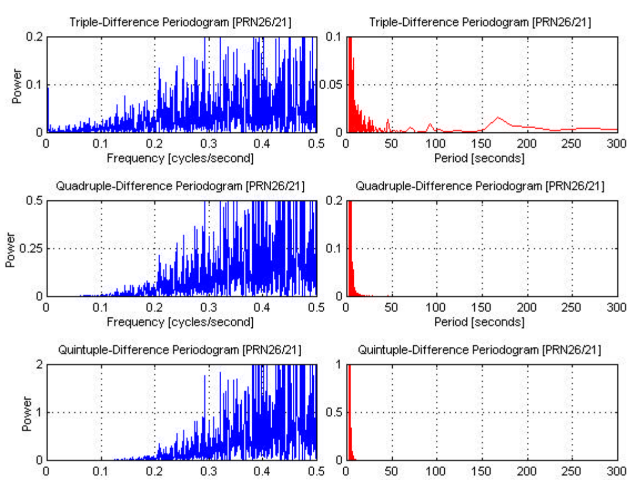

Figure 5. Cancellation of multipath (Test 1).

To obtain a time series with significant ionospheric delay, we took a time series from the Test 3 data sets. In static, long-baseline environments, we can expect that the effect of the ionospheric delay is still significant after double differencing. Furthermore, the effect of it increases at a low data rate (0.1 Hz). In addition, the selected time series has weak multipath (Figure 6).

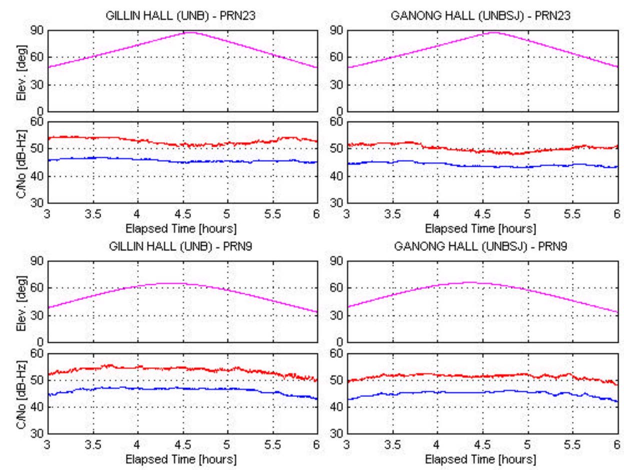

Figure 6. Elevation angle and C/N$_0$ for long-baseline weak multipath case (Test 3). The red line is the C/N$_0$ for L1(C1) and the blue line for L2(P2).

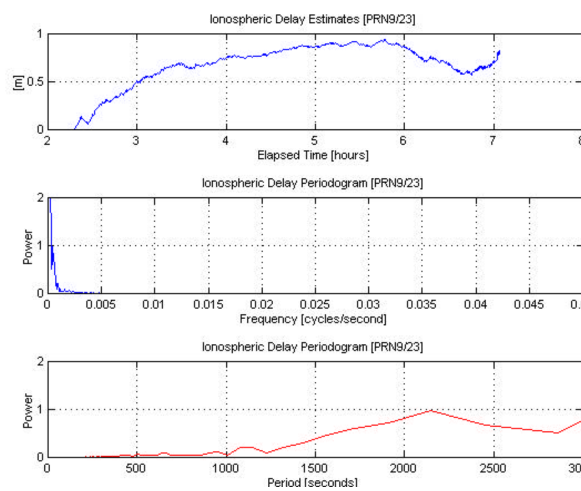

Figure 7. Example of a significant relative ionospheric delay in double-difference observations (Test 3).

We estimated the ionospheric delay for the time series using the L1 and L2 observations. In this case, the ionospheric delay estimates include true ionospheric delay, a constant bias due to L1 and L2 ambiguities, and the combined effects of the L1 and L2 observation noise.

Figure 7 shows justification for that: i.e., there are significant low-frequency components which reflect the effects of the ionospheric delay.

Now, we can confirm that the differencing process does cancel the effects of the ionospheric delay. Using the TD, QD and dQD time series in Figure 8, we investigated each time series in the frequency domain. As the order of the differencing increases, the frequency components of the ionospheric delay disappear as shown in Figure 9.

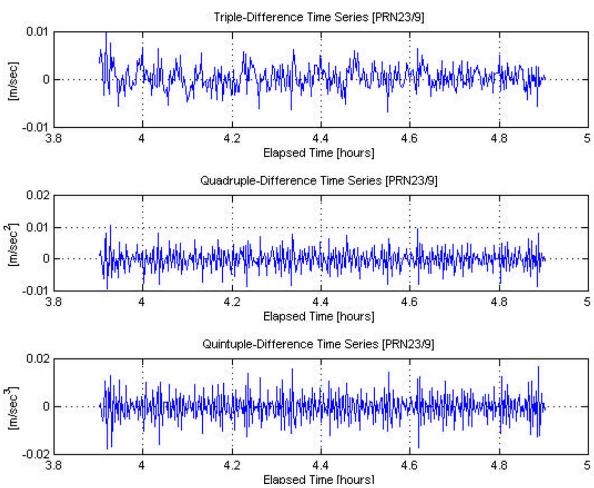

Figure 8. Time series of the ionospheric delay change estimates (Test 3).

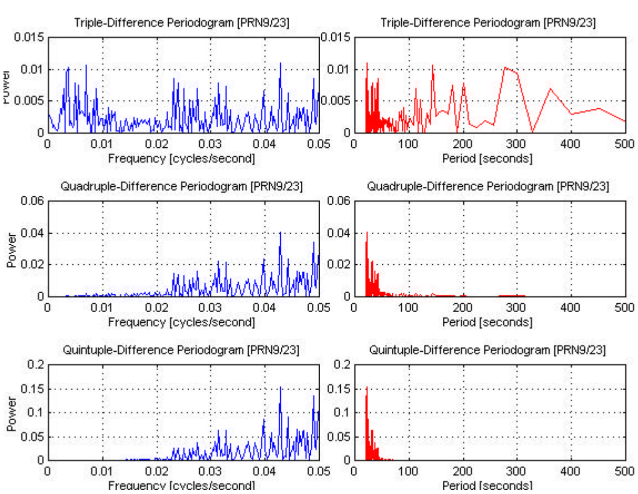

Figure 9. Cancellation of the ionospheric delay (Test 3).

With respect to the tropospheric delay and satellite orbit bias in Eq. (1), we expect similar cancellations to the ionospheric delay. Then, the remaining component whose cancellation in the differencing process we have to justify is the geometric range. In static mode, just satellite dynamics dominates the geometric range. On the other hand, in kinematic operations it is affected not only by satellite dynamics, but platform dynamics as well.

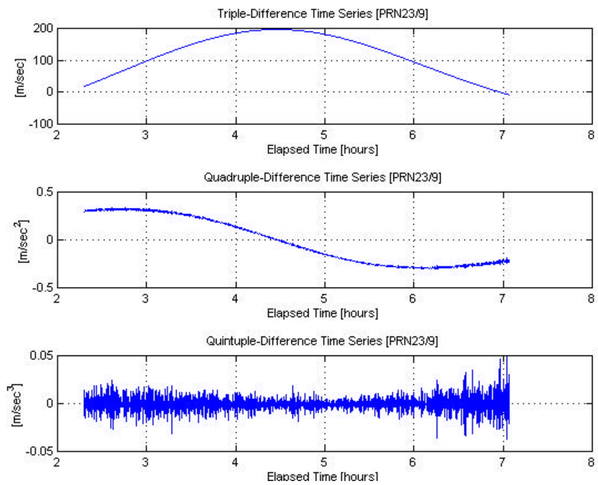

Figure 10. Time series of the geometric range (Test 3).

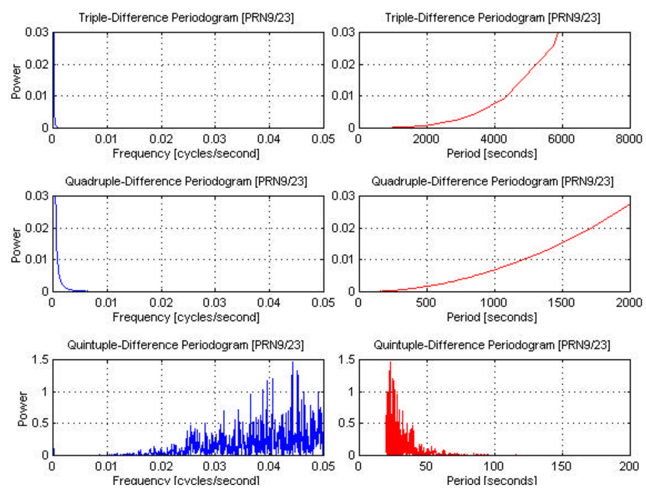

Figure 11. Cancellation of the geometric range (Test 3).

Figures 10 and 11 illustrate that the differencing process can easily cancel the effect of variations in the geometric range in static mode. However, it is not the case in kinematic mode. In other words, the effect of geometric range changes is not easily canceled in kinematic mode because the platform dynamics is apt to remove temporal correlation between the geometric ranges. Comparing the first plots in Figures 10 and 12, that of Figure 10 shows a quite smooth curve that the effect of the geometric range variations can be easily canceled in the differencing process while that of Figure 12 shows that cannot. Consequently, Figure 13 shows justification for the assertion that the differencing process cannot cancel the effect of the geometric range variations in kinematic mode.

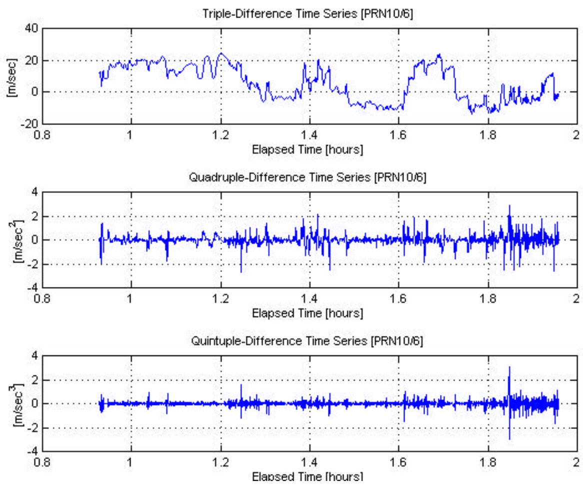

Figure 12. Time series of the geometric range (Test 4).

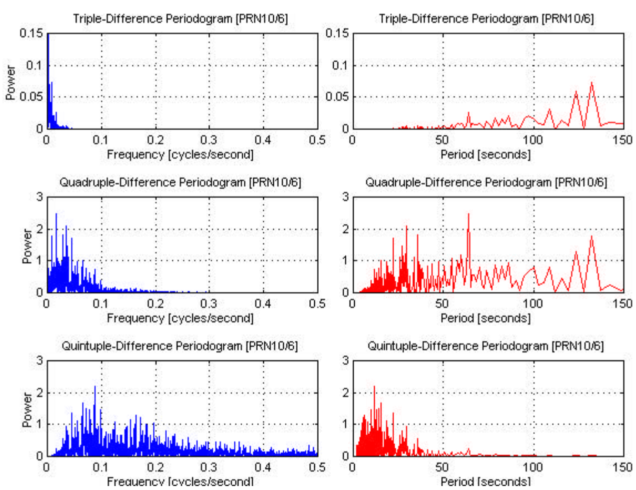

Figure 13. Effect of platform dynamics on the geometric range (Test 4).

Up to now, we have justified that biases can be canceled by differencing in the time domain. We have also identified that geometric range variation is a problematic component in our approach, particularly when the data is recorded at a low data rate (e.g., 1 Hz) in kinematic mode. Based on our justification, we estimated fully populated variance-covariance matrices for observation noise and compared the results with those of the C/N$_0$ approach. The following four figures (from Figures 14 to 17) show the results.

Several comments should be given to understand observation noise estimation figures: First, when we estimate observation noise using the C/N$_0$ approach (i.e., the top two plots in each figure), we must know at least two factors correctly – a conversion equation between the receiver's SNR output and the C/N$_0$, and the signal tracking loop bandwidths. We used the published SNR-to-C/N$_0$ conversion equation for the Ashtech Z-XII receiver. However, we did not have available the correct information for the signal tracking loop bandwidths. Therefore, we used an example value (the same value for L1, L2, C1 and P2) given in *Langley* [1997]. The effect of incorrect values for signal tracking loop bandwidths, however, just scales the true values. This means that the pattern of the original estimates holds. Second, when platform dynamics exist, our approach may not provide realistic estimates for observation noise depending on the data rate as illustrated above. To justify the effect of platform dynamics, we estimated observation noise using the time series of the geometry-free (GF) linear combinations for carrier-phase and pseudorange (i.e., the bottom two plots in each figure):

$$\begin{aligned} GF_L &= DD_{L1} - DD_{L2} \\ GF_P &= DD_{C1} - DD_{P2} \end{aligned} \qquad (10)$$

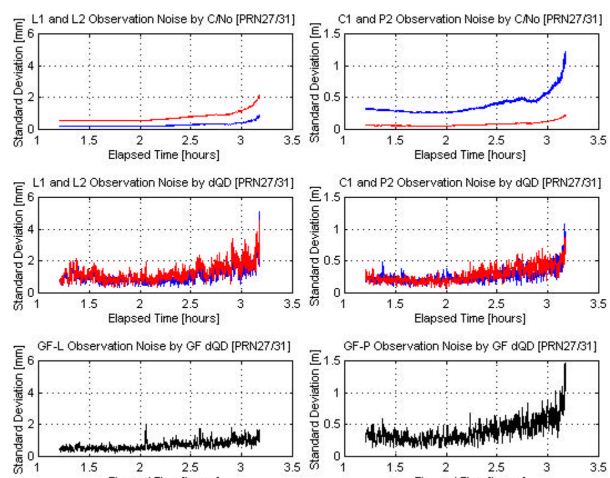

Figure 14. Observation noise estimation (Test 1). The red line is the estimates for L1(C1) and the blue line for L2(P2).

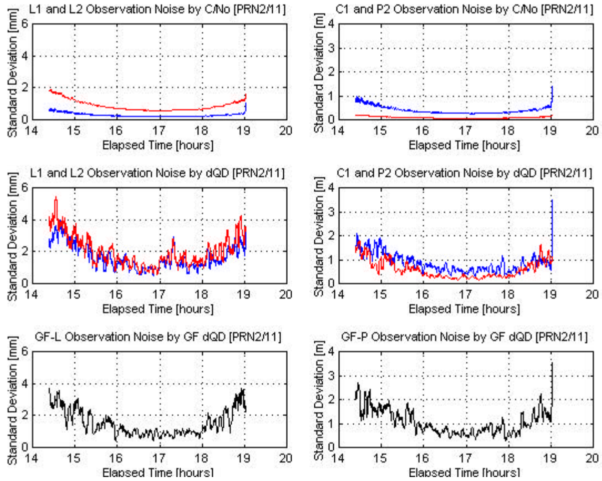

Figure 15. Observation noise estimation (Test 3). The red line is the estimates for L1(C1) and the blue line for L2(P2)

Figures 14 and 15 show the estimates of observation noise in static mode. Indeed, our approach provides realistic estimates either in short-baseline or long-baseline situations, at 1 Hz or 0.1 Hz data rates. Interestingly, we can see high cross-correlation between the observation types in both figures. Furthermore, the estimates of L1 and L2, C1 and P2 observation noise are not much different. At this moment, we cannot explain why we don't see a difference between L1 and L2, C1 and P2 noise levels. We will investigate this behaviour in our future work. As was expected, the estimates of our approach are not much different from those of the GF combination in static mode.

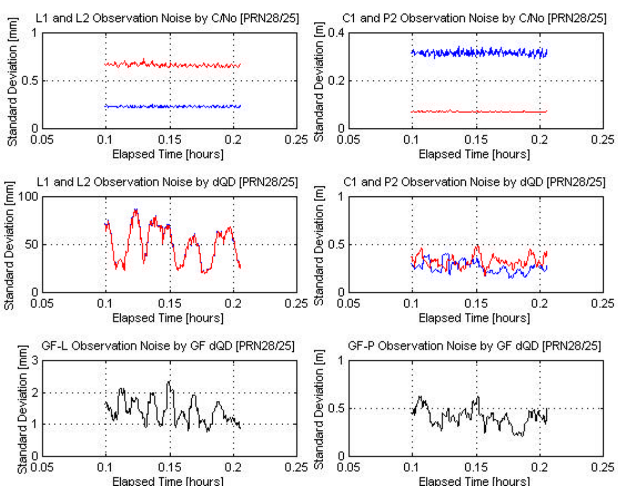

Figure 16. Observation noise estimation (Test 2). The red line is the estimates for L1(C1) and the blue line for L2(P2)

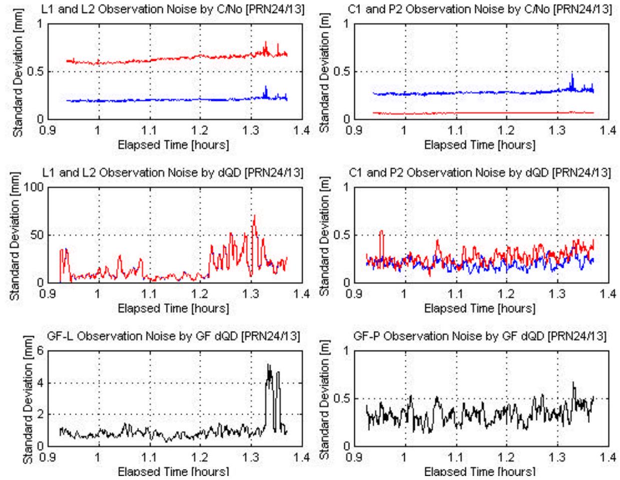

Figure 17. Observation noise estimation (Test 4). The red line is the estimates for L1(C1) and the blue line for L2(P2)

However, we can see the effects of platform dynamics in the estimates of our approach in kinematic mode (Figures 16 and 17): i.e., the estimates of our approach are highly affected by platform dynamics. Looking at the GF plots for carrier-phase observations, the effect of the platform dynamics is very clear. Meanwhile, it is not so clear for pseudoranges because the effects of platform dynamics are buried in the pseudorange observation noise.

Finally, we need to mention the effect of multipath in kinematic mode. It is not difficult to imagine that a moving platform can change the geometry of multipath. Depending on the platform dynamics, there may be rapid changes of the geometry, so that temporal correlation of multipath can be removed. This decorrelation cannot be canceled even in the GF linear combinations. The patterns in the GF plots illustrate that.

CONCLUSIONS

In our new approach, we have tried to overcome the three main problems of existing approaches in determining the stochastic model for GPS observations – lack of a fully populated variance-covariance matrix, missing temporal correlation, and no observation redundancy in long-baseline applications.

A general conclusion about our investigations is that temporal correlation in biases can be eliminated by differencing observations in the time domain as long as observation time series are highly time-correlated. This is the fundamental idea in our approach. Then, there might be a question: i.e., how can we obtain highly time-correlated observation time series? There is a general rule which can be understood in terms of the physics inherent in the differencing process: i.e., the data rate should be sufficiently high to make each component of the biases temporally correlated. If the effects of high-frequency (compared with the data rate) components of the biases are significant in the time series, the data rate should be increased to cancel the effects of such components in the differencing process. The significance of the conclusion is that there is just one degree of freedom for obtaining realistic noise estimates and that we can control it.

In static mode, conventional data rates (e.g., 1 Hz or 0.1 Hz) are sufficient to obtain realistic estimates for the observation noise. However, we have to increase the data rate in kinematic mode. In this case, the most problematic high-frequency components are the jerk of the geometric range and multipath due to the moving platform dynamics. Fortunately, high data rate receivers (e.g., 10-50 Hz) are available on the commercial market. If we can use such receivers, it is easy to implement a routine procedure to estimate observation noise in data processing software.

Another conclusion related to our future work is that we can consider a fusion of the C/N_0 approach and ours. For example, we can use our approach as a tuning process for the C/N_0 approach because our approach provides realistic noise estimates in static mode or at a high data rate. We can also use the C/N_0 approach as a monitoring process for our approach because our approach is degraded by platform dynamics while the C/N_0 approach is not.

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