Mitigation of GPS Carrier Phase Multipath Effects in Real-Time Kinematic Applications

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BIOGRAPHIES

Donghyun Kim is a post-doctoral fellow in the Department of Geodesy and Geomatics Engineering at the University of New Brunswick (UNB), where he has been developing a new on-the-fly (OTF) ambiguity resolution technique for long-baseline kinematic GPS applications and software for a gantry crane auto-steering system using the carrier-phase observations of high data rate GPS receivers. He has a B.Sc., M.S. and Ph.D. in geomatics from Seoul National University. He has been involved in GPS research since 1991 and is a member of the IAG Special Study Group "Wide Area Modeling for Precise Satellite Positioning".

Richard Langley is a professor in the Department of Geodesy and Geomatics Engineering at UNB, where he has been teaching since 1981. He has a B.Sc. in applied physics from the University of Waterloo and a Ph.D. in experimental space science from York University, Toronto. Prof. Langley has been active in the development of GPS error models since the early 1980s and is a contributing editor and columnist for GPS World magazine.

ABSTRACT

In some situations where short delay and low frequency multipath due to a moving platform itself and close-by reflectors is predominant, a parametric estimation approach based on functional models, which relate the multipath errors of the code and carrier tracking loops with multipath parameters, is useful in mitigating the effects of multipath in the code and carrier phase measurements.

We have carried out a feasibility study to investigate potential problems in the parametric estimation approach for multipath in real-time and kinematic applications. Based on our understanding of the approach using a multiple antenna-receiver system, we have tried to extend earlier work with several modifications such as different types of antenna-receiver system configurations, careful consideration of the antenna/receiver gain pattern, and the use of dual-frequency antennas and receivers, and so on.

We have found that parametric estimation for multipath is feasible in real-time and kinematic applications as long as the observability problem in the measurement sensitivity matrix of the multipath parameters is solved. Several concluding remarks and recommendations related to hardware and software system implementation are discussed from a practical point of view.

INTRODUCTION

For the last decade or so, it has been a continuing interest to mitigate the effects of multipath in GPS code and carrier phase measurements. From the simplest approach like optimal antenna location selection, to the most complicated receiver technology, a number of multipath mitigation techniques have been proposed by many groups from all over the world. In terms of the capability of real-time processing and kinematic applications, multipath mitigation by means of improved receiver technologies is preferable. However, as described in Braasch and Van Dierendonck [1999], all current receiver architectures have similar multipath performance in the presence of short delay multipath. In fact, recent receiver technologies have significantly improved mainly medium and long delay multipath performance in the pseudorange and carrier phase measurements [Van Dierendonck *et al.*, 1992; Townsend and Fenton, 1994; Townsend *et al.*, 1995; Moelker, 1997; Garin and Rousseau, 1997]. As long as the receiver (actually its antenna) is not in the vicinity of large obstacles, errors of 10 m or less are usual. In such situations, a parametric estimation approach based on functional models, which describe the relationship between multipath errors and signal delay due to multipath, is useful for real-time and kinematic applications.

Functional models which relate the multipath errors of the code and carrier tracking loops with multipath parameters have been well described [Van Nee, 1993; Braasch, 1996; Brodin, 1996; Braasch, 1997], where the primary multipath parameters are strength, delay, phase and phase-rate, all measured relative to the direct signals. Another functional model which links the signal-to-noise-ratio (SNR) values with the multipath parameters also has been used [Axelrad *et al.*, 1996; Comp and Axelrad, 1998; Reichert and Axelrad, 1999]. Recent research to integrate the functional models in parametric estimation procedures using a multiple antenna-receiver system was carried out by Ray [2000].

Based on our understanding of the parametric estimation approach using a multiple antenna-receiver system, we have tried to extend the technique to kinematic applications. The original concept was developed for a reference station operating in static mode. In this paper, we describe a preliminary feasibility study to investigate potential problems in implementing a system for real-time kinematic applications.

KINEMATIC APPLICATIONS UNDER SHORT DELAY AND LOW FREQUENCY MULTIPATH ENVIRONMENTS

Unlike high dynamics kinematic applications where the effects of multipath can be usually treated as quasi-random errors, there are some situations where short delay and low frequency multipath is predominant. We have been working on several projects which use carrier-phase measurements in real-time kinematic situations. In most such applications, our first interest is to resolve carrier-phase ambiguities either in short-baseline or long-baseline situations. However, we have often found that the effects of multipath in carrier phase measurements make it difficult to fix ambiguities correctly. Even if ambiguities are correctly resolved, we often had to reduce multipath in the measurements in order to improve positioning results.

An example of a short-baseline application is a gantry crane auto-steering system based on carrier phase measurements, which is being designed for a trading port (Figure 1). The following are the main features of the application:

1) Short-baseline application (moving cranes are usually within 1 km of a base station).
2) The system requires very high positioning accuracy (better than 2 cm horizontal accuracy at 99% confidence level).
3) Low-dynamics kinematic operation.
4) Real-time operation at a high data rate (10 Hz).
5) Multipath rich environment.

Fixing ambiguities is more or less an easy task in such a short-baseline application. However, in order to attain high accuracy positioning results, the effects of multipath in the measurements must be reduced or otherwise handled appropriately in designing the system. Signal reflections from the moving cranes themselves and close-by objects, which causes short delay and low frequency multipath, is the main concern in this case.

Fig. 1 – Gantry crane auto-steering system based on GPS carrier phase measurements.

An example of a long-baseline application is a bathymetric surveying project using a hydrographic sounding ship at Trois-Rivières, on the St. Lawrence River (Figure 2). The following are the main features of the application:

1) Long-baseline application (distance between a reference station in the Canadian Coast Guard DGPS/OTF network and the ship may reach over 100 km).
2) Low-dynamics kinematic operation.
3) Either post-processing or real-time operation.

Fixing ambiguities is not an easy task at all in such situations. Frequently, multipath makes it more difficult to resolve ambiguities correctly. We are interested in multipath due to close-by reflectors on the moving ship in this case.

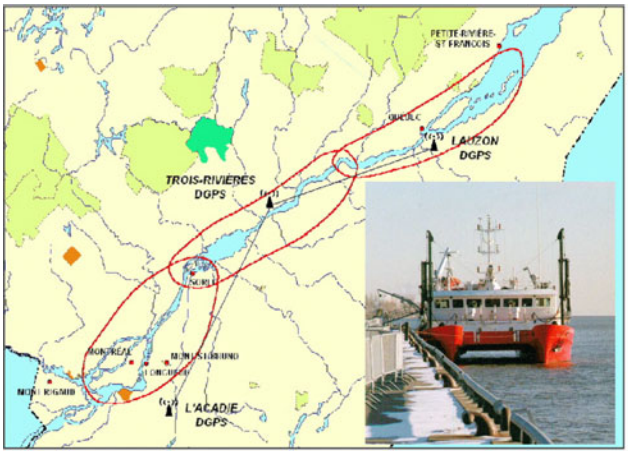

Fig. 2 – Bathymetric surveying in conjunction with high precision GPS positioning.

To summarize our interests in these applications, we need to handle reflections from the moving platform itself and close-by reflectors which cause short delay and low frequency multipath.

PARAMETRIC ESTIMATION OF MULTIPATH

In order to carry out parametric estimation for multipath, we must define functional models which relate the multipath errors of the code and carrier tracking loops with multipath parameters. Analysis of the code and carrier tracking loops is required for that purpose. We are not going to reiterate such procedures to obtain the functional models in this paper. Instead, we will take a brief look at the fundamentals of the approach from a practical point of view. Many assumptions are involved with the implementation of a multiple antenna-receiver system to estimate multipath parameters such as (see Ray [2000] for detail):

1) Multipath is spatially correlated within a small area where each antenna is under the same multipath environment.
2) Multipath in the pseudorange, carrier-phase and SNR measurements can be well approximated by the functional models including common multipath parameters.
3) A single virtual (or effective) reflector corresponding to the composite multipath signal is assumed.

Common multipath parameters in the functional models include the following:

1) Reflector-dependent parameters such as reflection coefficient, reflected signal's azimuth and elevation angle.
2) Antenna/receiver-dependent parameters such as antenna orientation and gain pattern.
3) Satellite-dependent parameters such as direct signal's azimuth and elevation angle.
4) Observation-type-dependent parameters such as reflected signal's phase and the ratio of the direct and reflected signals' autocorrelation functions.

To state the fundamentals of the approach, first of all, we have to obtain multipath observables from the pseudorange, carrier phase and SNR measurements. This can be easily carried out with a multiple-antenna configuration, each antenna feeding a separate receiver. For example, by single-differencing between receivers, most of the biases and errors such as the atmospheric delay, satellite orbit error, and satellite clock bias can be removed. If an external atomic frequency standard is available and feeds each of the receivers, the receiver clock bias also can be removed. The single-differenced ambiguities can be easily estimated in such situations because a multiple antenna configuration provides very short baselines (*e.g.*, a few tens of centimetres). The resulting observation equations are reduced to:

$$\begin{aligned}\Delta\Phi &= \Delta\rho + \Delta M_L + \Delta\varepsilon_L \\ \Delta P &= \Delta\rho + \Delta M_C + \Delta\varepsilon_C \\ \Delta C/N_0 &= \Delta s_0 + \Delta M_S + \Delta\varepsilon_S,\end{aligned} \tag{1}$$

where Φ, P and C/N_0 are the measured carrier phase (m), pseudorange (m) and carrier-to-noise power density ratio (dB-Hz), respectively; ρ is the geometric range from antenna phase centre to GPS satellite; M is the multipath error of the signal tracking loops which can be modelled by the functional models including the common multipath parameters (see the papers mentioned in the section "Introduction" for the derivation of the functional models); s_0 is a signal power factor; ε is measurement noise; subscripts "L", "C" and "S" represent carrier phase, pseudorange and C/N_0, respectively; and Δ is the single difference operator. With respect to the C/N_0 measurements expressed in equation (1), it is assumed that the effects of multipath are predominant. However, it should be noted that there may be differences due to other factors such as differences in preamp gain, cable loss, ground radiation, interference, *etc*. Ignoring such differences under the assumption of predominant multipath, the signal power factor s_0 becomes (see Ray [2000] for detail)

$$s_0 = 10 \cdot \log_{10}\left(\frac{R^2(\hat{\tau}_C)}{N_0}\right),$$
$$\because \quad C/N_0 = 10 \cdot \log_{10}\left(\frac{\bar{P}}{N_0}\right), \quad \bar{P} = R^2(\hat{\tau}_C) \cdot M_s, \tag{2}$$

where $R(\cdot)$ represents an autocorrelation function; $\hat{\tau}_C$ is the receiver estimate of the incoming signal code delay; N_0 is noise power; and $\bar{P}$ is the average received signal's power. Note that Δs_0 can be removed from equation (1) only if the value of ratio in parentheses is the same for each antenna-receiver system. In a practical sense, we had better not remove the antenna gain pattern factor in the functional model related to the C/N_0 measurements because real situations are far from ideal.

Among the terms in equation (1) the geometric range can be readily computed in static applications. However, it must be estimated simultaneously with the multipath parameters in kinematic mode. Therefore, we can find two different geometry and redundancy requirements in kinematic applications:

1) Satellite geometry and the number of satellites which are required to isolate the geometric ranges from the code and carrier phase measurements.
2) Antenna geometry, and the number of antennas and observation types which are required to estimate the multipath parameters using the multipath observables.

Since the geometric range should be estimated with the multipath parameters in kinematic applications, overall performance of the parametric estimation for multipath is to some extent affected by the accuracy of the geometric range estimates. In fact, the geometric range estimates are transformed into the relative antenna azimuths and elevation angles via antenna position estimates in implementing the functional models. A few centimetres of positioning error due to multipath (using the carrier phase measurements) can cause several degrees of relative antenna azimuth and elevation angle error because the distance between antennas is so short. In this respect, dual-frequency receivers are preferable in kinematic applications because we can reduce the effects of multipath and observation noise using an optimal inter-frequency carrier phase linear combination of the L1 and L2 observations in estimating antenna positions. For example, a generic inter-frequency carrier phase linear combination of the single-differenced L1 and L2 observations can be expressed as:

$$k_1\Delta\Phi_1 + k_2\Delta\Phi_2 = \Delta\rho + k_1(\Delta M_{L1} + \Delta\varepsilon_{L1}) + k_2(\Delta M_{L2} + \Delta\varepsilon_{L2})$$
$$k_1 = \frac{\alpha\lambda_2}{\alpha\lambda_2 + \beta\lambda_1}, \quad k_2 = \frac{\beta\lambda_1}{\alpha\lambda_2 + \beta\lambda_1}, \quad \text{and } \alpha, \beta \in \mathbb{R}, \tag{3}$$

where λ is the carrier wavelength and subscripts "1" and "2" represent L1 and L2 carrier phases, respectively. For all values of α and β $(\in \mathbb{R})$, the following holds:

$$k_1 + k_2 = 1. \tag{4}$$

Therefore, an optimal inter-frequency carrier phase linear combination of the L1 and L2 observations, which can reduce the effects of multipath and observation noise, can be found by solving

$$\min_{k1,k2}\left(KQK^T\right), \quad with \ k_1 + k_2 = 1, \tag{5}$$

where $K = [k_1 \quad k_2]$ and Q is the variance-covariance matrix of the L1 and L2 multipath observables. Given the variance-covariance matrix Q, it is easy to prove that the coefficients k_1 and k_2 reduce the combined effects of multipath and observation noise in the inter-frequency carrier phase linear combination of the L1 and L2 observations (compared with the effects of multipath and observation noise in either the L1 or L2 observations alone).

As usual in parametric estimation, the stochastic models of the measurements can also affect the overall performance of the parametric estimation for multipath. The stochastic models can be obtained through analysis of the code and carrier tracking loops. For practical reasons, however, we use a "differencing-in-time" approach described in Kim and Langley [2001] which corresponds to a high-pass filtering technique.

TEST SYSTEM SET-UP

At the beginning of our investigation of the parametric estimation approach, we thought that we could estimate the multipath parameters if we had enough measurement redundancy in terms of the number of observation types. This lead us to look for a particular receiver which provides the largest number of observation types. Figure 3 shows a test system set-up comprising two Ashtech Z-12 receivers, two NovAtel L1/L2 Pinwheel antennas, and one Temex Rb atomic frequency standard. The external frequency standard feeds 5 MHz signals to both receivers via a signal splitter. Two antennas were installed on one side of a rotating bar which simulates kinematic situations.

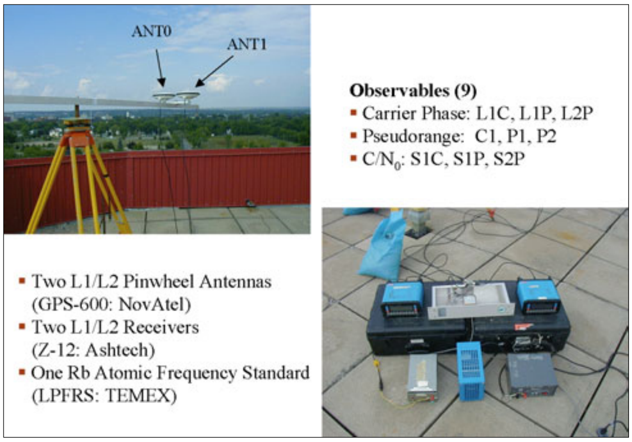

Fig. 3 – Test system set-up to estimate the multipath parameters.

The observation equations of the system can be expressed in a conventional matrix-vector form as:

$$\begin{bmatrix} L1C \\ L1P \\ L2P \\ C1 \\ P1 \\ P2 \\ S1C \\ S1P \\ S2P \end{bmatrix}_{SV} = \begin{bmatrix} \rho \\ \rho \\ \rho \\ \rho \\ \rho \\ \rho \\ 0 \\ 0 \\ 0 \end{bmatrix}_{SV} + \begin{bmatrix} \times & {\color{blue}\times} & {\color{red}\times} & \times & \times & 0 & 0 \\ \times & {\color{blue}\times} & {\color{red}\times} & \times & 0 & \times & 0 \\ \times & {\color{blue}\times} & {\color{red}\times} & \times & 0 & \times & 0 \\ \times & {\color{blue}\times} & {\color{red}\times} & \times & \times & 0 & 0 \\ \times & {\color{blue}\times} & {\color{red}\times} & \times & 0 & \times & 0 \\ \times & {\color{blue}\times} & {\color{red}\times} & \times & 0 & \times & 0 \\ \times & {\color{blue}\times} & {\color{red}\times} & \times & \times & 0 & \times \\ \times & {\color{blue}\times} & {\color{red}\times} & \times & 0 & \times & \times \\ \times & {\color{blue}\times} & {\color{red}\times} & \times & 0 & \times & \times \end{bmatrix}_{SV} \begin{bmatrix} \text{reflection coefficient} \\ \text{reflected signal Az} \\ \text{reflected signal Elev} \\ \text{reflected signal phase} \\ \text{reflection ratio (C1)} \\ \text{reflection ratio (P)} \\ \text{antenna gain factor} \end{bmatrix}_{SV},$$

(6)

where symbol "x" represents partial derivatives corresponding to the multipath parameters (see Reichert [1999] and Ray [2000] for details); and subscript "*SV*" indicates a certain satellite. Unfortunately, we found that there is an observability problem in the measurement sensitivity matrix of the multipath parameters, which means that not only measurement redundancy but also antenna geometry are required to carry out the parametric estimation. The second and third columns in the measurement sensitivity matrix in equation (6), which correspond to the reflected signal orientation (azimuth and elevation angle), are identical except for a scale factor. This means that we can estimate the distance between antennas and a virtual reflector using the system set-up but the orientation of the reflector cannot be determined uniquely.

To solve the observability problem, we set up a second test system comprising three L1/L2 antennas and receivers (Figure 4). For compatibility reasons, we used three NovAtel receivers (two OEM4s and one MiLLenium).

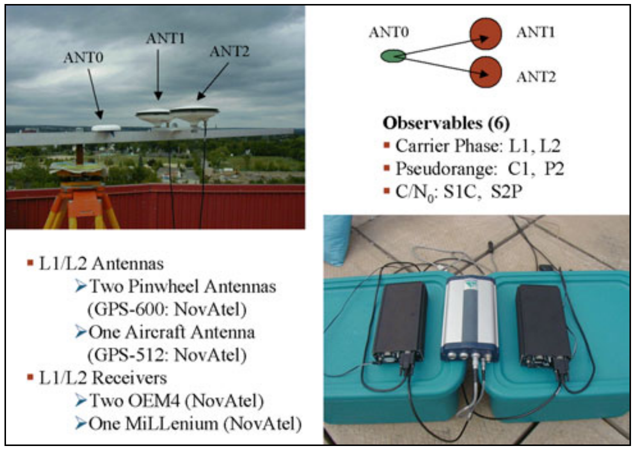

Fig. 4 – Test system set-up to estimate the multipath parameters.

System configuration pictured in Figure 4 is different from that proposed in the original approach: *i.e.*, different types of antennas and receivers were used. We did this on purpose to check potential problems with such a system configuration. Obviously, we could solve the observability problem by augmenting the single-differenced observations as seen in equation (7), since the antenna geometry with an additional antenna could discriminate between the reflected signal's azimuth and elevation angle.

$$\begin{matrix} \begin{bmatrix} L1 \\ L2 \\ C1 \\ P2 \\ S1 \\ S2 \end{bmatrix}_{0-1} \\ \begin{bmatrix} L1 \\ L2 \\ C1 \\ P2 \\ S1 \\ S2 \end{bmatrix}_{0-2} \end{matrix} = \begin{matrix} \begin{bmatrix} \rho \\ \rho \\ \rho \\ \rho \\ 0 \\ 0 \end{bmatrix}_{0-1} \\ \begin{bmatrix} \rho \\ \rho \\ \rho \\ \rho \\ 0 \\ 0 \end{bmatrix}_{0-2} \end{matrix} + \begin{bmatrix} \times & \times & \times & \times & \times & 0 & 0 & 0 \\ \times & \times & \times & \times & 0 & \times & 0 & 0 \\ \times & \times & \times & \times & \times & 0 & 0 & 0 \\ \times & \times & \times & \times & 0 & \times & 0 & 0 \\ \times & \times & \times & \times & \times & 0 & \times & 0 \\ \times & \times & \times & \times & 0 & \times & \times & 0 \\ \times & \times & \times & \times & \times & 0 & 0 & 0 \\ \times & \times & \times & \times & 0 & \times & 0 & 0 \\ \times & \times & \times & \times & \times & 0 & 0 & 0 \\ \times & \times & \times & \times & 0 & \times & 0 & 0 \\ \times & \times & \times & \times & \times & 0 & 0 & \times \\ \times & \times & \times & \times & 0 & \times & 0 & \times \end{bmatrix} \begin{bmatrix} \text{reflection coefficient} \\ \text{reflected signal Az} \\ \text{reflected signal Elev} \\ \text{reflected signal phase} \\ \text{reflection ratio (CA)} \\ \text{reflection ratio (P)} \\ \text{antenna gain factor (0-1)} \\ \text{antenna gain factor (0-2)} \end{bmatrix}$$

(7)

TEST RESULTS AND DISCUSSIONS

In order to investigate potential problems in the parametric estimation approach, we have tested it with data sets recorded in kinematic mode at 1 Hz data rate. As previously mentioned, NovAtel OEM4 and MiLLenium receivers were used to record dual-frequency data. Figure 5 shows the status of satellites observed during the approximately 20-minute test.

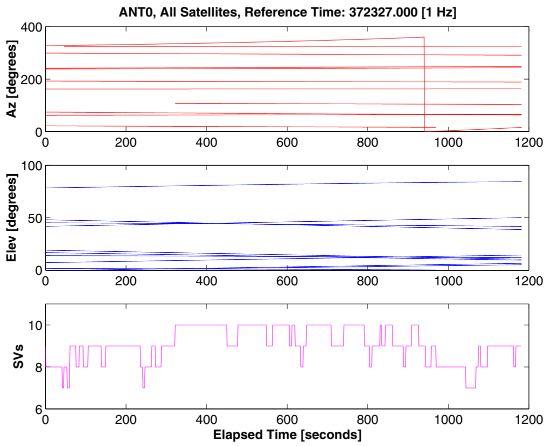

Fig. 5 – Azimuth, elevation angle and the number of satellites.

To estimate the multipath parameters, we implemented a Kalman filter with constant acceleration process noise models for various state variables. The choice of process noise models and values was empirical. The overall performance of multipath estimation was not that good as illustrated in Figures 6 and 7, where "Measurement" and "KF Estimation" represent the multipath observables in equation (1) and the computed values using the multipath parameter estimates, respectively.

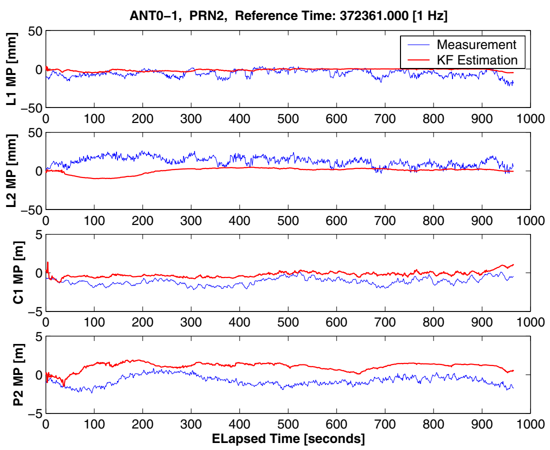

Fig. 6 – Multipath estimates of the single-differenced measurements between ANT0 and ANT1.

Since the initial results were not so good, we tried to figure out what was wrong with the hardware and software system implementation. A least-squares estimation based on single-epoch measurements was carried out for that purpose. To help figure out the problems, the multipath parameter estimates were tuned to the residuals of the L1 multipath for the ANT0-1 combination. This was easily done by setting a tolerance to the residuals of the L1 multipath. And then, all other multipath effects were predicted using the tuned multipath parameter estimates. Figures 8 through 11 illustrate the results.

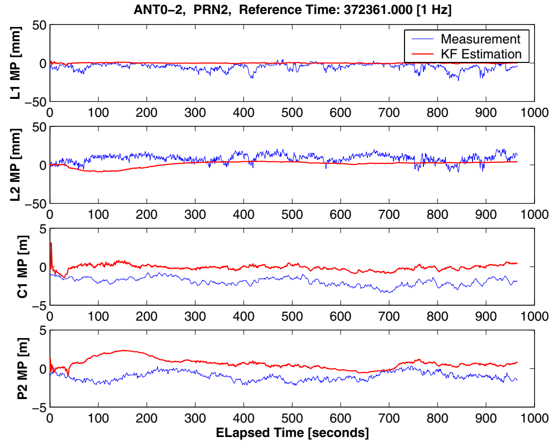

Fig. 7 – Multipath estimates of the single-differenced measurements between ANT0 and ANT2.

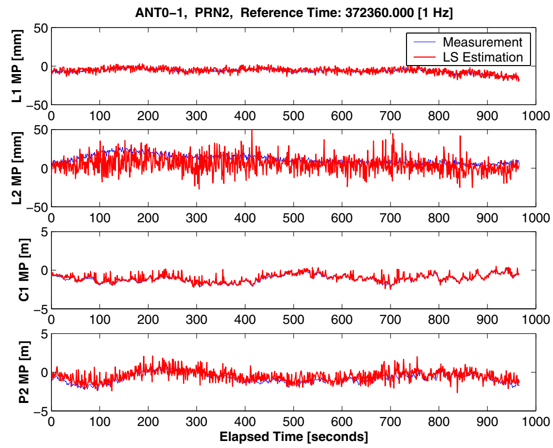

Fig. 8 – Multipath prediction of the single-differenced measurements between ANT0 and ANT1.

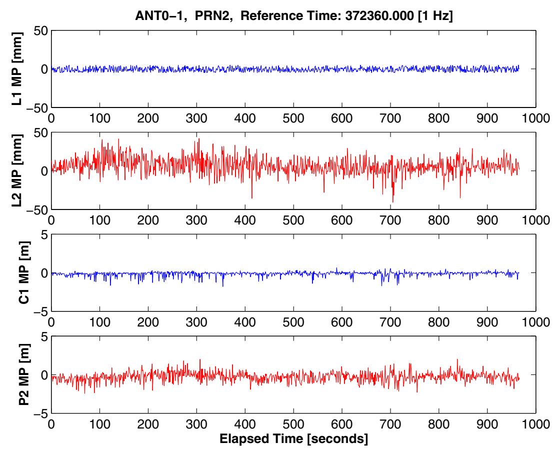

Fig. 9 – Multipath prediction residuals.

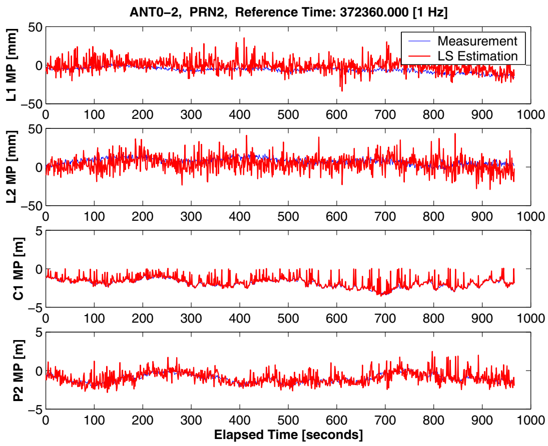

Fig. 10 – Multipath prediction of the single-differenced measurements between ANT0 and ANT2.

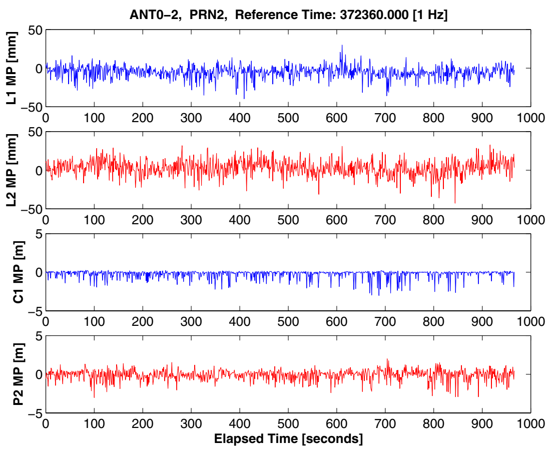

Fig. 11 – Multipath prediction residuals.

We can see two problems in Figures 8 through 11:

1) Compared with the first and third panels in Figures 8 and 9, those in Figures 10 and 11 show noisier and poorer performance. This reveals somehow that one assumption in the approach (that is, each antenna is under the same multipath environments.) may not be satisfied. Note that we used different types of antennas and receivers and this might have had an effect on the technique's performance.
2) Compared with the first and third panels in each figure, the second and forth panels show noisier and poorer performance. This reveals somehow that the functional models to predict (estimate) the L2 and P2 multipath may not be correct. Note that the second frequency observables are based on a specific codeless technique.

We have encountered another problem related to the antenna/receiver gain pattern. With respect to the L1/C1 C/N_0, it seems to carry reasonable multipath information in terms of correlation among antennas and noise level. However, L2/P2 C/N_0 values are noisy. Furthermore, there are many missing data (*i.e.*, zero values) due to signal dropouts in the measurements. Figures 12 and 13 illustrate such cases.

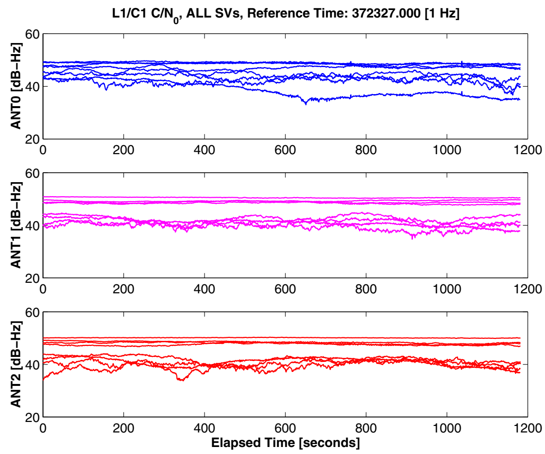

Fig. 12 – C/N_0 values for L1/C1.

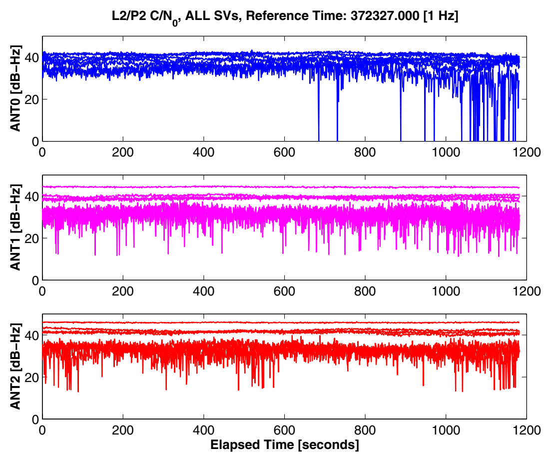

Fig. 13 – C/N_0 values for L2/P2.

Based on our investigation for the initial parametric estimation, we temporarily excluded the problematic observables (*e.g.*, P2 and S2) from the observation equations for the purpose of investigation. Then, we tried the parametric estimation again using the Kalman filer. As a result, we obtained more or less improved results (compared with those in Figures 6 and 7) as illustrated in Figures 14 and 15.

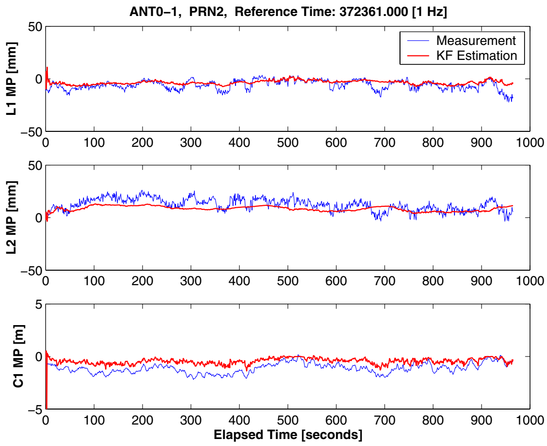

Fig. 14 – Multipath estimates of the single-differenced measurements between ANT0 and ANT1 using the L1, L2, C1 and S1 observables.

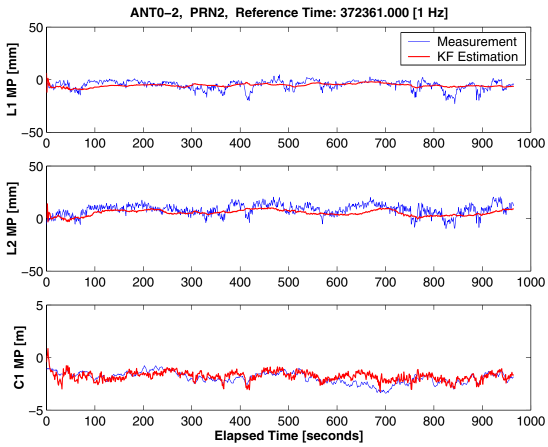

Fig. 15 – Multipath estimates of the single-differenced measurements between ANT0 and ANT2 using the L1, L2, C1 and S1 observables.

CONCLUDING REMARKS

We have carried out a feasibility study to investigate potential problems in the parametric estimation approach for multipath in real-time and kinematic applications. We have followed most of the fundamental ideas of a multiple antenna-receiver system which was originally proposed for a reference station operating in static mode. In order to extend the concept's work scope, we have investigated several modifications such as the use of different types of antenna-receiver system configurations, careful consideration of the antenna/receiver gain pattern, and the use of dual-frequency antennas and receivers, and so on.

We have found that parametric estimation for multipath is feasible for real-time and kinematic applications as long as the observability problem in the measurement sensitivity matrix of the multipath parameters is solved. However, we have encountered several practical problems in hardware and software system implementation as follows:

1) We must verify the functional models to predict (estimate) multipath in the second frequency observables: *i.e.*, L2, P2 and S2. In this respect, analysis of the code and carrier tracking loops for the second frequency observables is required.
2) We had better use the same type of antennas and receivers. Since the parametric estimation approach assumes that each antenna-receiver system is in the same multipath environment, an appropriate hardware system configuration is a minimal requirement.
3) From a geometrical point of view, an additional fourth antenna-receiver system will provide an ideal system configuration. Although two baselines formed by three antennas can discriminate the orientation of a virtual reflector, there may exist two intersection points which satisfy the functional models.

We have used a dual-frequency antenna-receiver system in our approach for two reasons:

1) The first reason was to improve the accuracy of relative antenna orientation estimates in real-time and kinematic situations because overall performance of the parametric estimation for multipath is affected by the accuracy of antenna orientation.
2) The second reason was to take advantage of the second frequency observables in increasing measurement redundancy and subsequently, to reduce the number of antennas and receivers.

Finally, further investigations will be carried out to obtain more convincing results from our approach. For that purpose, we consider introducing a strong reflector with a known geometry instead of relying on natural multipath.

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