A Reliable Approach for Ambiguity Resolution in Real-Time Long-Baseline Kinematic GPS Applications

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BIOGRAPHIES

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ABSTRACT

Resolving the GPS carrier-phase ambiguities has been a continuing challenge for sub-centimeter-level high-precision GPS positioning. In kinematic and long-baseline applications, the challenge turns out to be even greater due to the substantial problems in the observation time series – the decorrelation of biases, the quasi-random behavior of multipath and the correct interpretation of receiver system noise (or observation noise) for observations conducted in kinematic mode – which can be ignored to some degree in static and short-baseline applications. As baseline lengths grow longer, eventually, these problems will make it difficult to get reliable ambiguity solutions in kinematic applications.

We have found that the problems related to kinematic long-baseline applications can be handled in an optimal way when a particular generalized procedure is adopted in the observations processing scheme. The generalized procedure includes: a functional model which takes into account all significant biases; a stochastic model which is derived directly from the observation time series; a quality control scheme which handles cycle slips (or outliers); and a parameter-estimation scheme which includes a simultaneous ambiguity search process. The prototype approach described in this paper follows a generalized procedure for use when constraining external observations (such as those provided by an external atomic clock, inertial navigation system (INS) and so on) are not available. For each stage of the procedure, the new concepts of our approach are explained and some preliminary test results are given.

INTRODUCTION

It has been a continuing challenge to determine and fix the GPS carrier-phase ambiguities, especially for long-baselines. Moreover, the challenge is even greater for kinematic GPS applications. Generally, the difficulty in solving the ambiguities is due to the decorrelation of biases in the GPS observations. As is well known, the GPS observations at the base and remote stations will be influenced by different atmospheric effects and satellite orbit bias as the baseline length between the stations gets longer. Furthermore, when the pseudorange observations are incorporated with the carrier-phase observations, multipath can be the dominant error source which makes it difficult to solve the ambiguities because of its quasi-random behaviour over a relatively short time span. In kinematic situations, it is not easy to model the receiver system noise since the dynamics of a moving platform may mask some aspects of the receiver system noise which usually can be well modeled statistically by an elevation-angle dependent function.

To obtain optimal solutions in the least-squares estimation, a functional (or deterministic) and a stochastic model should be specified correctly. A functional model describes the relationship between observations and unknown parameters while a stochastic model represents the noise characteristics of the observations. Actually, the challenge that we face in long-baseline kinematic applications is how to correctly specify the models without ignoring the problems mentioned above; i.e., the decorrelation of biases, the quasi-random behavior of multipath and the receiver system noise for observations conducted in kinematic mode. In this case, the problem related to the functional model is that the number of unknown parameters is greater than that of the observations, when constraining external observations such as those provided by an external atomic clock, inertial navigation system (INS) and so on are not available. Furthermore, it turns out to be very difficult to specify a correct stochastic model if we opt for a simpler functional model by ignoring certain parameters because of the residual effects of these parameters as well as the dynamics of a moving platform. As a fundamental problem in processing the GPS observations, we also face a quality control issue; i.e., how do we implement a robust cycle-slip (or outlier) handling routine? Especially for long-baseline kinematic applications, this issue turns out to be another challenge.

Basically, our approach in attempting to meet these challenges follows a generalized procedure which consistently keeps track of the noted problems in long-baseline kinematic applications. The prototype approach described in this paper is based on the case when dual-frequency GPS observations are available. In situations where external observations are also available, the approach can integrate the additional information without undue complexity and ultimately, improve system reliability.

Considerations for a Reliable Approach

As has been experienced, the stochastic model is typically more difficult to handle than the functional model when considering a reliable approach for long-baseline kinematic applications. Assuming that the functional model includes all significant unknown parameters (e.g., those associated with atmospheric effects, satellite orbit bias, multipath and so on) except for receiver system noise (i.e., antenna noise, cable loss and receiver noise; see *Langley* [1997]), we can deal with the stochastic model more easily. In this case, the problems associated with the stochastic model are: cross correlation (between different observation types), time correlation (between epochs), spatial correlation (between channels), elevation-angle dependence and the error probability distribution [*Tiberius et al.*, 1999]. Basically, we assume that all parameters describing the stochastic modeling problems can be calibrated in the laboratory. As long as the functional model is correct, these parameters can be used at a remote site without tuning. However, it should be noted that the elevation-angle dependence of the system noise often varies with the particular kinematic situation. The elevation-angle dependence of the system noise is induced mainly by the receiver antenna's gain pattern, with other factors such as atmospheric signal attenuation. The elevation angle is normally computed with respect to the local geodetic horizon plane at the antenna phase center regardless of the actual orientation of the antenna. Accordingly, the relationship between antenna gain and the signal elevation angle may be difficult to assert when the antenna orientation is changing which can happen often in kinematic situations.

If we use a functional model which includes all significant unknown parameters, we will face a problem in conventional least-squares estimation or Kalman filtering; i.e., the singularity or observability problem. Although this problem can be handled at second-hand by a parameter transformation method to reduce the number of the unknown parameters [*Jin*, 1996], an inherent difficulty still remains: 1) uncertainty of the parameter estimators is hardly improved because there may be no surplus redundancy. In using a Kalman filter, redundancy can be improved by additional information from the dynamic model of the filter. However, it is not easy to come by an accurate dynamic model. Introducing an inaccurate dynamic model into the filter brings about the divergence of the Kalman filter [*Fitzgerald*, 1971]. It is possible to partially compensate for the effects of inaccuracy of the dynamic model by increasing the intensity of process noise assumed by the filter. However, uncertainty of the parameter estimators will not be improved much in that case; 2) the parameter estimators can be biased to some degree due to the higher-frequency components of the unknown parameters (e.g., ionospheric scintillation, jerk and so on). In the context of signal processing, this problem is related to the sampling rate and any aliasing [*Ifeachor and Jervis*, 1993]. For example, if we have an observation time series recorded at a one second sampling interval (i.e., 1 Hz sampling rate), the time series can contain information in a frequency band up to 0.5 Hz (i.e., the Nyquist frequency). If the effects of any higher-frequency components above 0.5 Hz are significant, the time series will include aliased signals. This will result in biased state estimators after all. Ionospheric scintillation and jerk due to the platform dynamics can bring about the problem in GPS long-baseline kinematic applications. To obtain a wide bandwidth which includes higher-frequency components, we have to increase the sampling rate. By how much should we increase it? The quick answer is enough to remove the effect of aliasing in order to get unbiased parameter estimators. In other words, we need the observation time series obtained at an appropriate sampling rate for a specific application.

KALMAN-FILTER-BASED AMBIGUITY SEARCH PROCESS

There can be many approaches to the question: Which strategy will be preferable for handling GPS observations in long-baseline kinematic applications? However, in terms of implementation, our answer is a Kalman filter approach combined with an ambiguity search method which can deal with both the functional and stochastic models in an optimal way (Fig. 1).

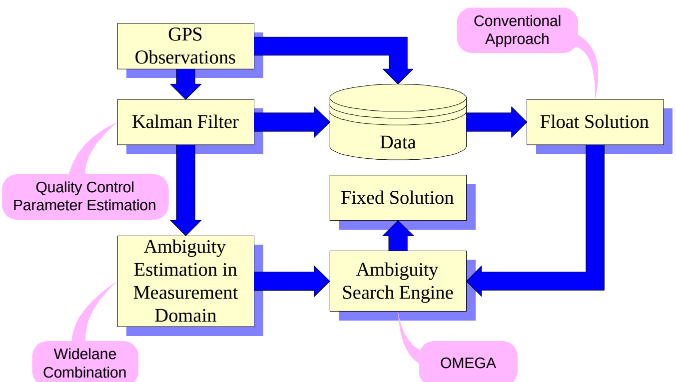

Fig. 1. Functional block diagram to fix the GPS ambiguities in long-baseline kinematic applications.

Parameter Estimation using a Kalman Filter

A Kalman filter approach can efficiently implement quality control schemes such as cycle-slip handling (i.e., cycle-slip detection, identification and adaptation), and that the state estimators of the filter can be used at second-hand in the ambiguity search process as long as the state estimators are not biased. However, fundamental concerns related to its implementation are: 1) How do we reduce the number of unknown parameters in the filter state vector? 2) How do we ensure the observability of the given system model under the rank-deficiency condition? 3) Which implementation method is most efficient?

Basically, the problem is that the number of unknown parameters is much greater than that of the observables. This is an inherent problem of carrier-phase applications and turns out to be a substantial one in such an approach as ours which tries to estimate all the bias parameters (so far, all except for the multipath in the carrier-phase observations). For example, we use the following geometry-free observation model [*Teunissen*, 1997] which includes the dual-frequency observations for each double-difference satellite-receiver pair:

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ P_1 \\ P_2 \end{bmatrix}_k = \begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & -g & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & g & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} L \\ I \\ b_1 \\ b_2 \\ B_1 \\ B_2 \\ n_1 \\ n_2 \end{bmatrix}_k + \begin{bmatrix} e_1 \\ e_2 \\ h_1 \\ h_2 \end{bmatrix}_k \quad (1)$$

where L stands for the combined effect of a priori (or assumed) geometric range, satellite orbit bias and tropospheric delay $(L = r + s + t)$; I for the L1 ionospheric delay; b for the multipath in carrier phases; B for the multipath in code pseudoranges; n for the ambiguities (in distance units); a constant $g = (l_2 / l_1)^2$; subscripts "1" and "2" correspond to L1 and L2, P1 and P2 (or C/A and P2), respectively. The subscript k represents a current observation epoch.

To reduce the number of unknown parameters, the double differencing scheme is used in Eq. (1). In addition, dual-frequency carrier phases (L1 and L2) and code pseudoranges (P1 and P2, or C/A and P2) is used to increase observation redundancy. Furthermore, the unknown parameters are transformed to ensure the observability of the given system model. A separate Kalman filter is implemented for each double-difference time series because its programming and stochastic modeling are easier. As a result, we form the following state vector:

$$\tilde{\mathbf{x}}_k = \begin{bmatrix} \tilde{L} & \cdots & \tilde{I} & \cdots & \tilde{B}_1 & \cdots & \tilde{B}_2 & \cdots & \tilde{n}_1 & \tilde{n}_2 \end{bmatrix}_k^T \quad (2)$$

with

$$\begin{aligned} \tilde{L} &= L + \frac{1}{g-1}(g\,b_1 - b_2) + \frac{1}{g-1}(g\,B_1'^0 - B_2'^0) \\ \tilde{I} &= I + \frac{1}{g-1}(b_1 - b_2) - \frac{1}{g-1}(B_1'^0 - B_2'^0) \\ \tilde{B}_1 &= B_1' - B_1'^0 \\ \tilde{B}_2 &= B_2' - B_2'^0 \\ \tilde{n}_1 &= n_1 + \frac{1}{g-1}\left[-(g+1)B_1'^0 + 2B_2'^0\right] \\ \tilde{n}_2 &= n_2 + \frac{1}{g-1}\left[-2g\,B_1'^0 + (g+1)B_2'^0\right] \end{aligned} \quad (3)$$

and

$$
\begin{aligned}
B_1' &= B_1 + \frac{1}{g-1}\left[-(g+1)b_1 + 2b_2\right] \\
B_2' &= B_2 + \frac{1}{g-1}\left[-2g\,b_1 + (g+1)b_2\right] \\
B_1'^0 &= B_1^0 + \frac{1}{g-1}\left[-(g+1)b_1^0 + 2b_2^0\right] \\
B_2'^0 &= B_2^0 + \frac{1}{g-1}\left[-2g\,b_1^0 + (g+1)b_2^0\right],
\end{aligned}
\tag{4}
$$

where superscript "0" stands for the initial (at the start of observations) bias value;; "$\cdots$" for the higher-order time derivatives of the parameters. The "~" symbol indicates a transformed parameter. Therefore, the transformed observation model becomes

$$
\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ P_1 \\ P_2 \end{bmatrix}_k = \begin{bmatrix} 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -g & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & g & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{L} \\ \tilde{I} \\ \tilde{B}_1 \\ \tilde{B}_2 \\ \tilde{n}_1 \\ \tilde{n}_2 \end{bmatrix}_k + \begin{bmatrix} e_1 \\ e_2 \\ h_1 \\ h_2 \end{bmatrix}_k \tag{5}
$$

It should be noted that each transformed parameter of Eq. (2) includes a true parameter value, the carrier-phase multipath on L1 and L2, constant initial-multipath ($B_1'^0$ and $B_2'^0$). The constant initial-multipath can be separated from the parameter estimators in the ambiguity search process (see section '*Ambiguity Search Process*') but the carrier-phase multipath is so far difficult to estimate in this approach as long as we cannot use additional observations such as the signal-to-noise ratio (SNR) or alternatively the carrier-to-noise-power-density ratio (C/N_0) for the carrier-phase observations.

Due to the definition of $\tilde{B}_1$ and $\tilde{B}_2$ in Eq. (3), we can initialize the transformed observation model. After initialization, time-dependent state parameters $(\tilde{L}, \tilde{I}, \tilde{B}_1 \text{ and } \tilde{B}_2)$ can be estimated because the ambiguity parameters $(\tilde{n}_1 \text{ and } \tilde{n}_2)$ are time-independent. It is evident that there is no surplus redundancy in the observation model. Therefore, we need to introduce dynamic models to improve uncertainty of the parameter estimators.

For each transformed parameter, we can introduce the corresponding dynamic model. For example, a constant-acceleration dynamic model may be assumed for $\tilde{I}$. In this case, the third-order time derivative of $\tilde{I}$ can be modeled as a zero-mean white noise process with a constant spectral density $q_{\ddot{\tilde{I}}}$ (m^2/s^5). Then, we can form the following dynamic model for $\tilde{I}$:

$$
\begin{bmatrix} \tilde{I} \\ \dot{\tilde{I}} \\ \ddot{\tilde{I}} \end{bmatrix}_k = \begin{bmatrix} 1 & \Delta t_k & \frac{1}{2}\Delta t_k \\ & 1 & \Delta t_k \\ & & 1 \end{bmatrix} \begin{bmatrix} \tilde{I} \\ \dot{\tilde{I}} \\ \ddot{\tilde{I}} \end{bmatrix}_{k-1} + \mathbf{e}_k \tag{6}
$$

with

$$
\begin{aligned}
E\left[\mathbf{e}_k\right] &= \mathbf{0} \\
E\left[\mathbf{e}_k \mathbf{e}_l^T\right] &= q_{\ddot{\tilde{I}}} \begin{bmatrix} \frac{1}{20}\Delta t_k^5 & \frac{1}{8}\Delta t_k^4 & \frac{1}{6}\Delta t_k^3 \\ & \frac{1}{3}\Delta t_k^3 & \frac{1}{2}\Delta t_k^2 \\ Sym. & & \Delta t_k \end{bmatrix}
\end{aligned}
\tag{7}
$$

where $\mathbf{e}$ is a process noise vector and $\Delta t_k = t_k - t_{k-1}$. In the same way, we can introduce dynamic models for $\hat{L}$ and $\hat{B}$. For the ambiguity parameter $\hat{n}$, we have to assign zero process noise because it is time-independent. Note that the parameter estimators determined in this way can be biased in some cases as mentioned previously in the section '*Considerations for a Reliable Approach*'. In addition, we will not get unbiased parameter estimators if cycle slips are not handled perfectly.

Quality Control

Since we do not consider a cycle slip as an unknown parameter in the functional model, we have to detect and remove it from the observations. If we fail to do that, the Kalman filter parameter estimators will be biased after all. Basically, we have used a cycle-slip handling procedure similar to that of *Teunissen* [1990a]; i.e., the DIA (Detection, Identification and Adaptation) procedure based on the Kalman filter prediction residuals. However, we have found that the procedure does not work as well as expected in kinematic situations. This problem is due to the dynamics of a moving platform and eventually, related to sampling rate. To avoid the effect of the platform dynamics, we can increase the process noise of the dynamic model of the filter; i.e., by assigning a very large variance to the time-dependent state parameters. However, this may result in unnoticed outliers due to an increase in process noise and subsequently, an increase in the variance-covariance matrix of the predicted state (see *Teunissen*, 1990b).

To fortify the procedure against platform dynamics, we use a masking technique based on a logical intersection of necessary and sufficient conditions for cycle-slip detection and identification. When a cycle-slip happens, we can see a certain spike in the quadruple-difference (obtained by differencing consecutive triple-difference observations) time series. We consider the spike pattern as a necessary condition for cycle-slip detection and identification. In some respects, this approach is similar to the wavelet transform technique to detect cycle slips [*Collin and Warnant*, 1995]. As a conventional approach incorporated within a Kalman filter, we can use prediction residuals to detect a cycle slip. However, this should be used carefully because the prediction residuals are very sensitive to the dynamics of a moving platform and the sampling rate of the observations. Another approach given in *Kee et al.* [1997] is the use of the ionospheric-delay drift estimators. However, this also should be used carefully because there are cases when a cycle slip cannot be detected such as when cycle slips of the same magnitude (in distance units) occur simultaneously on L1 and L2, not to mention the very obvious case when cycle slips in both carrier phases cancel each other in the ionosphere-free combination (i.e., $\frac{1}{77}c1-\frac{1}{60}c2=0$, where *c1* and *c2* represent cycle slips on L1 and L2 in cycle units). Though the former case is quite exceptional, according to our investigations, the case occurs when the millisecond jump happens in a certain receiver clock but the time tag is inconsistent with it. Nevertheless, in a wide sense, we consider that these two approaches – prediction residuals and ionospheric-delay drift estimators – provide sufficient conditions for detecting cycle slips.

So far, we have found that the performance of this procedure is almost perfect as far as cycle-slip detection and identification are concerned. However, cycle-slip adaptation (i.e., removing a cycle slip from the observations) should be executed carefully because the magnitude of the cycle slip must be determined correctly to remove it. If we try to determine the magnitude of a cycle slip using the Kalman filter prediction residuals, we may introduce a new bias in the observations. As a simple strategy to avoid this problem, we can reset the Kalman filter state vector whenever a cycle slip is detected and identified.

Receiver System Noise Estimation

If the stochastic model is not correct, it will affect the computed parameter estimators to some degree. In the case where we increase the process noise of the dynamic model of the filter to avoid the effect of the platform dynamics, stochastic modeling turns out to be crucial because the filter depends mainly on the measurement noise. As was mentioned already, we had better not use the elevation-angle dependent function for the stochastic model for data collected from moving platforms. An alternative, and potentially more powerful, approach can be derived directly from measurements of the quality of each pseudorange and carrier-phase observation. This information is contained in the SNR (or alternatively in the C/N_0). This value determines, in part, how well the receiver's tracking loops can track the signals and hence (to a large degree) how precisely the receiver obtains pseudorange and carrier-phase observations [*Langely*, 1997]. Although the potential merits of using the SNR information as a stochastic modeling scheme was already discussed by *Talbot* [1988], a comprehensive examination of the technique has only taken place recently [*Hartinger and Brunner*, 1998; *Barnes et al*., 1998]. Although some GPS receiver manufacturers provide SNR values in their data streams, meaningful SNR values are not easy to come by (see *Collins and Langley*, *1999*). Furthermore, multipath signals can adversely impact the receiver SNR depending on whether the direct and reflected signal components reaching the receiver combine constructively or destructively [*Cox et al.*, 1999]. As another approach incorporated within a Kalman filter, a covariance-matrix estimation method based on measurement filtering residuals can be considered [*Wang*, 1999]. In this approach, we have to assign a very large process noise to the time-dependent state parameters so that the filter operates only on the measurement noise. If our goal is just estimating receiver system noise, this approach provides an optimal estimate for that. If this is not the case, say, if we want to get accurate state parameter estimators as well as receiver system noise, we cannot simply assign a very large process noise to the time-dependent state parameters.

The following concept represents another experimental approach which can be derived directly from the observation time series under a simple assumption. This approach is independent of the Kalman filtering. We use the quintuple-difference (differencing consecutive quadruple-difference observations after deleting cycle-slip spikes) time series to estimate the receiver system noise for observations conducted in kinematic mode. We have chosen this approach because the quadruple-difference time series are already obtained for quality control as described in section '*Quality Control*'. Therefore, this approach can be implemented without undue complexity. In this approach, we assume that the effects of the unknown parameters (except the receiver system noise) are removed in the quintuple-difference time series. For example, consider the L1 quintuple-difference carrier-phase time series

$$\dddot{\Phi}_1 = \dddot{r} + \dddot{t} + \dddot{s} - \dddot{I} + \dddot{b}_1 + \dddot{n}_1 + \dddot{e}_1, \tag{8}$$

where Φ_1 is the L1 double-difference observable. Using the one-dimensional Taylor series including higher-order time derivatives for each unknown parameter, we have

$$\begin{aligned} S(t) &= S(t_0) + S'(t_0)(t-t_0) + \tfrac{1}{2}S''(t_0)(t-t_0)^2 \\ &\quad + \tfrac{1}{6}S'''(t_0)(t-t_0)^3 + R(t), \end{aligned} \tag{9}$$

where S represents each unknown parameter and R is a remainder term known as the Lagrange remainder. Assuming that the observation time interval $(t-t_0)$ is the same as Ä for the time series, we have the following quintuple-difference:

$$\dddot{S}(t_3) = S(t_3) - 3\cdot S(t_2) + 3\cdot S(t_1) - S(t_0) \\ = S'''(t_0)\Delta^3 + \Sigma_R(t_3)\,, \tag{10}$$

where $\Sigma_R(t_3)$ is the quintuple-difference for the remainder *R*. Substituting Eq. (10) into (8) gives

$$\dddot{\Phi}_1(t_3) = \left[\sum_{\forall S} S'''(t_0)\right]\Delta^3 + \sum_{\forall S}\left[\Sigma_R(t_3)\right] + \dddot{e}_1(t_3)\,, \tag{11}$$

where

$$\sum_{\forall S} S'''(t_0) = \left[\, r''' + t''' + s''' - I''' + b_1''' + n_1'''\right](t_0)\,. \tag{12}$$

If the effect of the terms in the right-hand side of Eq. (12) is small enough to be ignorable and/or the sampling rate (1/Ä) is high in Eq. (11), and if the effect of the second term in the right-hand side of Eq. (11) (i.e., the effect of the quintuple-difference for the remainder *R*) is also small enough to be ignorable, we can get an acceptable inference as:

$$\dddot{\Phi}_1 \approx \dddot{e}_1\,. \tag{13}$$

However, the underlying assumption in Eq. (13) is apt to be violated in high-dynamic environments because r''' (the jerk of the geometric range) can be so predominant that it may not be eliminated in the quintuple-difference time series. This fact urges us to use the observations obtained at an appropriate sampling rate for specific applications; e.g., 1-4 Hz sampling rate in hydrographic applications.

Ambiguity Search Process

Using the estimators of the state vector, we can transform the original carrier-phase double-difference observations to those to be used for the ambiguity search process. The purpose of this transformation is to reduce the number of unknown parameters at the ambiguity search step. However, there can be some cost to pay for this transformation (i.e., the receiver system noise is increased and time-correlated). We use an ionosphere-free transformation to reduce this cost:

$$\Phi_1 + \hat{I} = L + \frac{1}{g-1}b' + n_1 - \frac{1}{g-1}B_{12}'^{0} + e_1' \\ \Phi_2 + g\,\hat{I} = L + \frac{1}{g-1}b' + n_2 - \frac{g}{g-1}B_{12}'^{0} + e_2'\,, \tag{14}$$

where $b' = g\,b_1 - b_2$ and $B_{12}'^{0} = B_1'^{0} - B_2'^{0}$. As a matter of fact, we have found that the transformed observations are similar to the ionosphere-free linear combination but have smaller receiver system noise. The time-correlated receiver system noise can be estimated using the variance-covariance matrix which is obtained adaptively from the Kalman filter. Using the transformed observation for each double-difference observation, we have the augmented observation equations as

$$\mathbf{\ddot{O}}_1 + \hat{\mathbf{I}} - \left(\tilde{\mathbf{n}}_0 + \hat{\mathbf{o}}_0\right) = \mathbf{Ax} + \mathbf{\ddot{E}}_1\mathbf{N}_1 + \mathbf{s} + \frac{1}{g-1}\mathbf{b}' - \frac{1}{g-1}\mathbf{B}_{12}'^{0} + \mathring{\mathbf{a}}_1' \\ \mathbf{\ddot{O}}_2 + g\,\hat{\mathbf{I}} - \left(\tilde{\mathbf{n}}_0 + \hat{\mathbf{o}}_0\right) = \mathbf{Ax} + \mathbf{\ddot{E}}_2\mathbf{N}_2 + \mathbf{s} + \frac{1}{g-1}\mathbf{b}' - \frac{g}{g-1}\mathbf{B}_{12}'^{0} + \mathring{\mathbf{a}}_2'\,, \tag{15}$$

where **x** includes unknown baseline components and the residual tropospheric delay; **A** is the corresponding design matrix for **x**; **N** is the ambiguity vector (in cycle units); $\mathbf{\ddot{E}}$ is the corresponding design matrix for **N**; **s** is the satellite orbit bias vector; $\mathbf{b}'$ is the carrier phase multipath vector; $\mathbf{B}_{12}'^{0}$ is a constant initial multipath vector; $\tilde{\mathbf{n}}_0$ and $\hat{\mathbf{o}}_0$ are the initial estimates of the geometric range and the tropospheric delay, respectively. In practical implementation of Eq. (15), we assume that the carrier-phase multipath is ignorable and precise satellite orbit is available. In addition, we use the UNB3 tropospheric delay prediction model [*Collins and Langley*, 1997a] to compute $\hat{\mathbf{o}}_0$. Furthermore, we use a design matrix derived from a differential mapping function to estimate the residual tropospheric delay [*Collins and Langley*, 1997b]. As can be understood in looking at Eq. (15), the ambiguities cannot be separated from the parameters $\mathbf{B}_{12}'^{0}$ because they are also constant. This problem can be solved using the widelane combination of the estimators $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ in distance units. From Eq. (3), we can formulate the widelane combination as

$$\hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_2 = \left(\Lambda_1\mathbf{N}_1 - \Lambda_2\mathbf{N}_2\right) - \mathbf{B}_{12}'^{0}\,. \tag{16}$$

As the Kalman filter converges, the widelane combination verges to a constant quickly. This means that the parameters $\mathbf{B}_{12}'^{0}$ can be determined in the ambiguity search process, where $\mathbf{N}_1$ and $\mathbf{N}_2$ are given as known values, as long as the widelane combination, $\hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_2$, converges.

For the ambiguity search process, we use the independent-ambiguity-search approach [*Hatch*, 1990]. Since there remain four unknown parameters in the observation equations after the observations are transformed, we always have eight search levels (four search levels for $\mathbf{N}_1$ and $\mathbf{N}_2$, the L1 and L2 ambiguities, respectively) regardless of the number of double-difference observations. In this case, the search space may be enormous even if a small search window is used. This means that the ambiguity search process may be so time-consuming that it is not appropriate for a real-time system. In order to overcome this problem, we use an efficient ambiguity search engine, namely OMEGA (Optimal Method for Estimating GPS Ambiguities) as described by *Kim and Langley* [1999a and 1999b].

TEST RESULTS

In order to illustrate the performance of our technique, it has been tested with data sets recorded in static and kinematic modes during the UNB spring 2000 surveying camp. The static data set (UNB-STA) was recorded at a ten-second sampling interval at Gillin Hall located at UNB in Fredericton for 20 hours from 21:30 on 2 May 2000 and simultaneously at Ganong Hall at UNB's Saint John campus. The distance between the stations was more than 80 km. The kinematic data set (UNB-KIN) was recorded at a one-second sampling interval on board a vehicle traveling in the downtown area of Saint John and surrounding highways for 2 hours on 3 May 2000. Ashtech Z-XII receivers were used to record dual-frequency data. We also have used a marine kinematic data set (CCG-KIN) to illustrate one problem related to cycle slips. The dual-frequency data were recorded at a one-second sampling interval on board a hydrographic sounding ship on the St. Lawrence River on 22 October 1998 and simultaneously at one reference station (Trois-Rivières DGPS) in the Canadian Coast Guard (CCG) DGPS and OTF network. Ashtech Z-XII and Trimble 4000 SSI receivers were used at the reference station and on the ship, respectively.

We compared two approaches for estimating receiver system noise; i.e., the C/N_0 observations and the quintuple-difference time series. Estimation models for receiver system noise using the C/N_0 observations are given by *Langley* [1997].

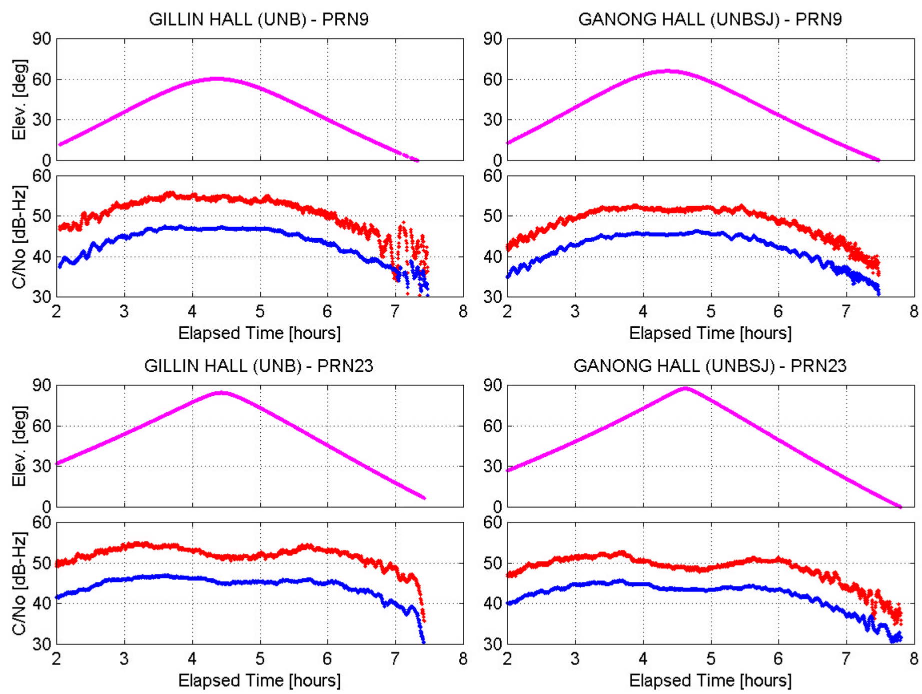

Fig. 2. Elevation angle and carrier-to-noise-power-density ratio (C/N_0) in static mode (UNB-STA). Red line is the C/N_0 observations for L1(C/A) and blue line for L2(P).

Fig. 2 shows a typical example for the relationship between the satellite elevation angle and the C/N_0 observations in static mode; i.e., the C/N_0 observations show a good elevation-angle dependency. Generally, the signal is weak and apt to be contaminated by multipath at low elevation angles. In kinematic situations, however, the signal can be contaminated by the platform dynamics as well as multipath even at high elevation angles (Fig. 3). This is why it may not be advisable to use an elevation-angle dependent function in estimating the receiver system noise, especially in kinematic mode. Based on this fundamental analysis, we estimated the receiver system noise using both approaches. We found that both approaches have similar performance in static mode although the estimates of the C/N_0 observations gave smooth curves while those of the quintuple-difference time series produced more realistic results (Fig. 4). In kinematic mode, the estimates of the quintuple-difference time series showed poor performance for L1 and L2 (Fig. 5). This was mainly due to the platform dynamics; i.e., the residual effect of r''' was so predominant that it was not eliminated in the quintuple-difference time series. We can see this fact clearly when we use the geometry-free combinations ($\Phi_1 - \Phi_2$ and $P_1 - P_2$); i.e., we can get realistic estimates of the receiver system noise from the geometry-free combinations because they are free from the platform dynamics (Fig. 6).

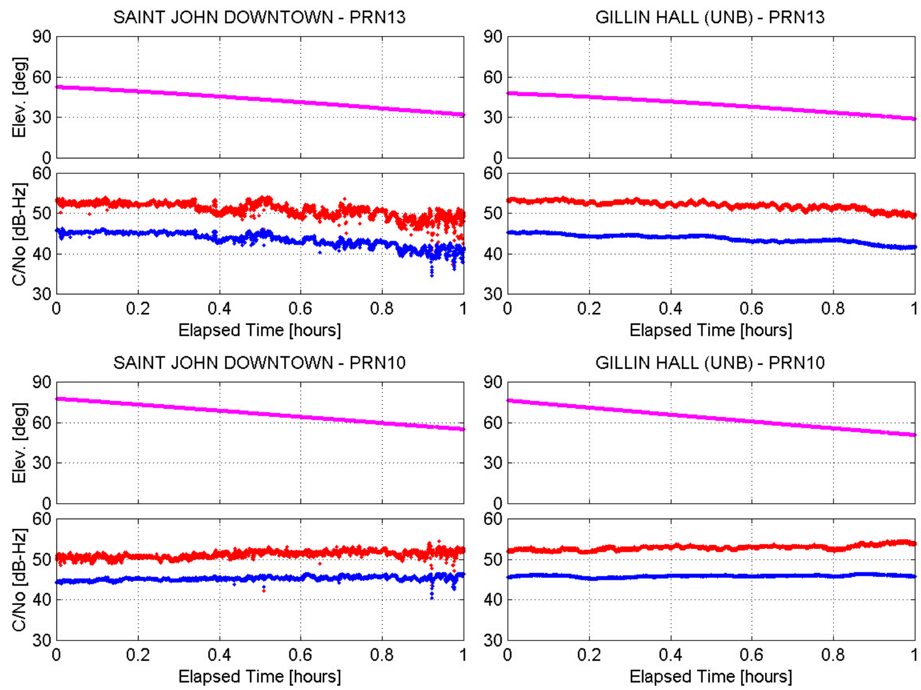

Fig. 3. Elevation angle and carrier-to-noise-power-density ratio (C/N_0) in kinematic mode (UNB-KIN). Red line is the C/N_0 observations for L1(C/A) and blue line for L2(P). The plots of Saint John downtown display the kinematic data.

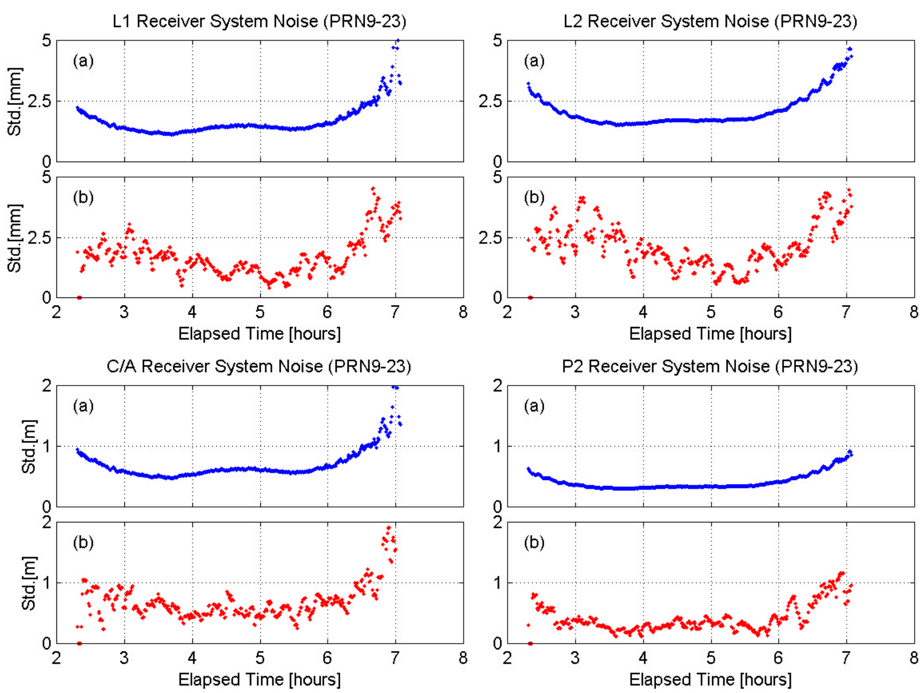

Fig. 4. Double-difference receiver system noise estimation in static mode (UNB-STA): (a) the C/N_0 approach and (b) the quintuple-difference approach. Y-axis (std.) represents the standard deviation (1 ó) of the estimators.

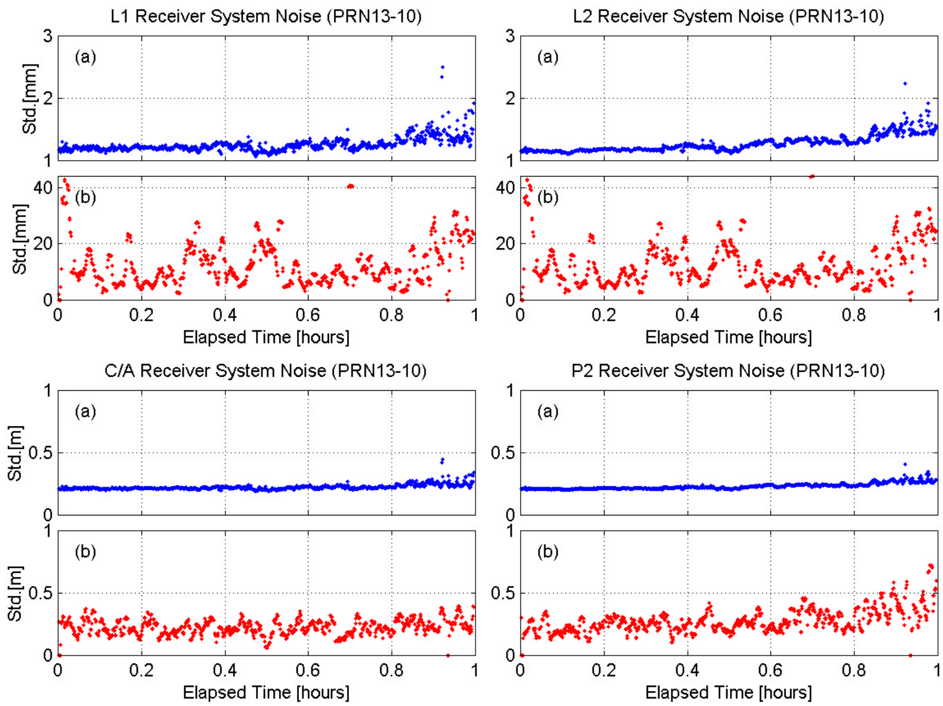

Fig. 5. Double-difference receiver system noise estimation in kinematic mode (UNB-KIN): (a) the C/N_0 approach and (b) the quintuple-difference approach. Y-axis (std.) represents the standard deviation (1 ó) of the estimators.

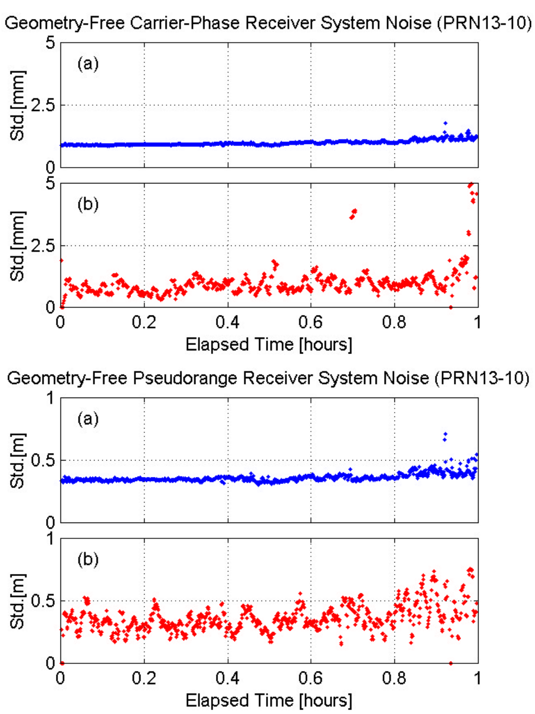

Fig. 6. Double-differnece receiver system noise estimation using the geometry-free combination in kinematic mode (UNB-KIN): (a) the C/N_0 approach and (b) the quintuple-difference approach.

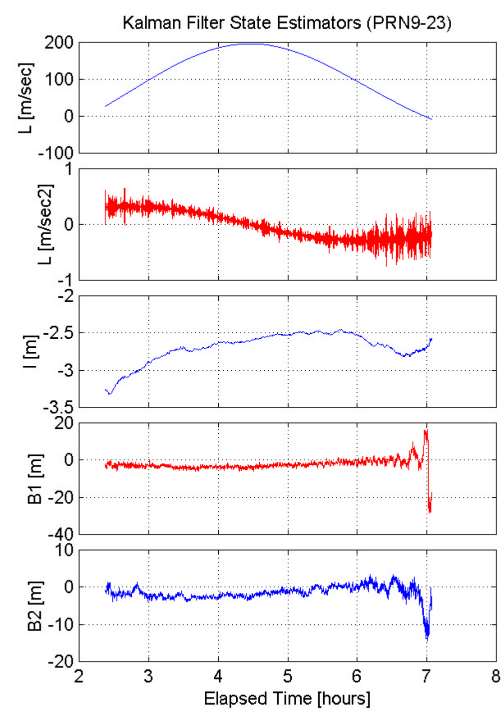

Fig. 7. Kalman filter parameter estimators for the double-difference time series (UNB-STA) of PRN9 and PRN23 in static mode.

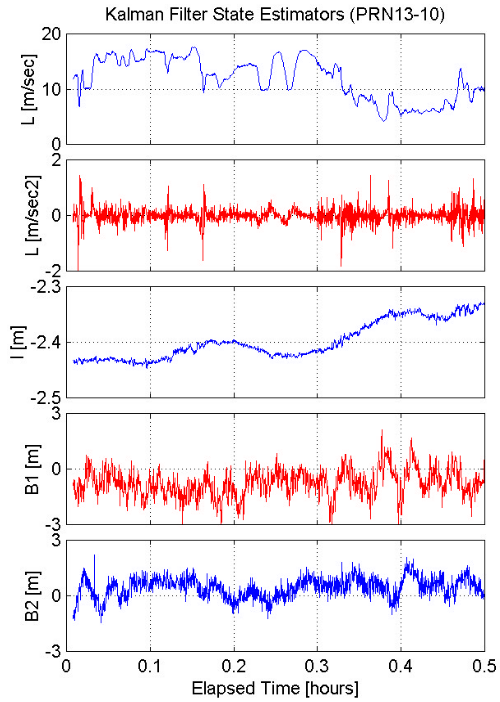

Fig. 8. Kalman filter parameter estimators for the double-difference time series (UNB-KIN) of PRN13 and PRN10 in kinematic mode.

Figure 7 and 8 show examples of the performance of the Kalman filter for the data sets (i.e., UNB-STA and UNB-KIN) obtained at the UNB spring 2000 surveying camp. Note that each parameter estimator includes the carrier-phase multipath on L1 and L2, the constant initial-multipath, and the receiver system noise. The most significant difference between the parameter estimators and true parameter values is an offset along the y-axis due to the constant initial-multipath. In Fig. 7, we can see clearly the effect of multipath after an elapsed time of 6 hours. Compare with Fig. 2. Furthermore, we can get good insight into the dynamic situations of the vehicle in Fig. 8. Although our investigations for the performance of the Kalman filter have shown good results, there might be a concern about the current approach: i.e., we need to increase observation redundancy. So far, there is no surplus redundancy in the observation model. As a result, we need to improve the accuracy of the initial estimators for the state parameters as well as the dynamic model in order to obtain accurate state parameter estimators. We will work on this issue in near future.

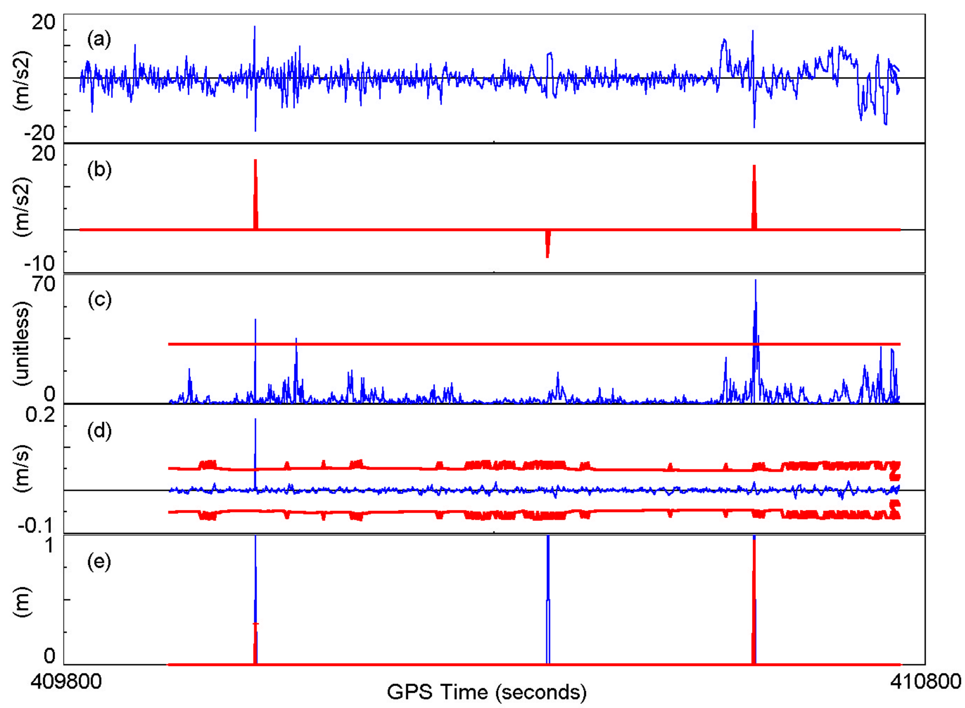

Fig. 9. Example of cycle-slip detection and identification procedures (CCG-KIN: PRN15&30): (a) L1 Quadruple-difference time series; (b) Cycle-slip candidates detected by spikes; (c) Cycle-slip candidates detected by the Kalman filter prediction residuals (95% confidence level); (d) Cycle-slip candidates detected by the ionospheric-delay drift estimators (95% confidence level); and (e) Masking results (cycle-slip identification).

Figure 9 illustrates the cycle-slip detection and identification procedures. We used the marine kinematic data set (CCG-KIN) to explain one problem related to the procedures. The performance of these procedures (Fig. 9e) is greatly improved compared with the conventional approaches as shown in Fig. 9c and 9d. For example, Fig. 9e shows perfect cycle-slip detection and identification results (i.e., there were two cycle slips on the observation time series used). On the other hand, Fig. 9c shows that the approach using the Kalman filter prediction residuals falsely detected cycle slips at certain epochs. If we set the confidence level lower than 95%, the results will be even worse. Furthermore, Fig. 9d shows that the approach using the ionospheric-delay drift estimators did not detect a cycle slip at a certain epoch where, in fact, a cycle slip did occur.

CONCLUDING REMARKS

We have developed a prototype approach to solve the ambiguity fixing problems in long-baseline kinematic applications. We focused mainly on the procedures to attain a reliable approach for such applications in this paper. The main feature of the technique, which may differ from other approaches, is that the system takes into account the problems of handling the decorrelation of biases, the quasi-random behavior of multipath and the receiver system noise in kinematic mode all at the same time within the functional and stochastic models for the GPS observations. In other words, we do not simply ignore these problems and hope their effects are averaged out. Instead, all the bias parameters and the receiver system noise (except multipath in the carrier-phase observations, so far) are estimated while a software process for quality control of the observations is proceeding.

The first step in designing a system for long-baseline kinematic applications is to define an appropriate functional model which takes into account all significant biases. If we take a simplified model, so that if the effects of the residuals due to the missing parameters are significant, the parameter estimators will be biased. Unfortunately, this is the usual case in long-baseline kinematic applications. The second step is to define an accurate stochastic model which represents the noise characteristics of the observations. If the functional model was defined appropriately, the effects of the observation residuals reflect mainly the receiver system noise. As long as the effects of the observation residuals are concerned with the receiver system noise, we can find good approximation methods for the noise: i.e., the C/N_0 observations and the quintuple-difference time series. The final step is to define a parameter estimation scheme. If we have accurate functional and stochastic models, it is not difficult to come by accurate parameter estimators. However, we face another challenge in this step; i.e., there is no surplus redundancy in the observation model. We will tackle this problem in our future work.

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REFERENCES

Barnes, J. B., N. Ackroyd and P. A. Cross (1998). “Stochastic modeling for very high precision real-time kinematic GPS in an engineering environment.” *Proceedings of the F.I.G. XXI International Congress*, Commission 6, Engineering Surveys, Brighton, U.K., 19-25 July, pp. 61-76.

Collin, F. and R. Warnant (1995). “Application of the wavelet transform for GPS cycle slip correction and comparison with Kalman filter.” Manuscripta Geodaetica, Vol. 20, No. 3, pp. 161-172.

Collins, J.P. and R.B. Langley (1997a). “A tropospheric delay model for the user of the Wide Area Augmentation System.” Final contract report prepared for Nav Canada, Department of Geodesy and Geomatics Engineering Technical Report No. 187, University of New Brunswick, Fredericton, N.B., Canada.

Collins, J.P. and R.B. Langley (1997b). "Estimating the residual tropospheric delay for airborne differential GPS positioning." *Proceedings of ION GPS-97*, Kansas City, MO, 16-19 September 1997, pp. 1197-1206.

Collins, J. P. and R. B. Langley (1999). “Possible weighting schemes for GPS carrier phase observations in the presence of multipath.” Final contract report for the U.S. Army Corps of Engineers Topographic Engineering Center, No. DAAH04-96-C-0086 / TCN 98151, March, 33 pp.

Cox, D. T., K. W. Shallberg and A. Manz (1999). “Definition and analysis of WAAS receiver multipath error envelopes.” *Navigation: Journal of the Institute of Navigation*, Vol. 46, No. 4, Winter, pp. 271-282.

Euler, H.-J. and H. Landau (1992). “Fast GPS ambiguity resolution on-the-fly for real-time application.” *Proceedings of Sixth International Geodetic Symposium* *on Satellite Positioning*, Columbus, Ohio, 17-20 March, pp. 650-659.

Fitzgerald, R. J. (1971). "Divergence of the Kalman filter." *IEEE Transactions on Automatic Control*, Vol. AC-16, No. 6, pp. 736-747.

Hatch, R. (1990). "Instantaneous ambiguity resolution." *Proceedings of KIS'90*, Banff, Canada, 10-13 September, pp. 299-308.

Hartinger, H. and F. K. Brunner (1998). "Attainable accuracy of GPS measurements in engineering surveying." *Proceedings of the F.I.G. XXI International Congress*, Commission 6, Engineering Surveys, Brighton, U.K., 19-25 July, pp. 18-31.

Ifeachor, E. C. and B. W. Jervis (1993). *Digital Signal Processing: A Practical Approach*. Addison-Wesley Publishing Co., Workingham, England.

Jin, X. X. (1996). "Theory of carrier adjusted DGPS positioning approach and some experimental results." Thesis. Delft University of Technology, Delft University Press, Delft, The Netherlands.

Kee, C., T. Walter, P. Enge and B. Parkinson (1997). "Quality control algorithms on WAAS wide-area reference stations." *Navigation: Journal of the Institute of Navigation*, Vol. 44, No. 1, Spring, pp. 53-62.

Kim, D. and R. B. Langley (1999a). "An optimized least-squares technique for improving ambiguity resolution performance and computational efficiency." *Proceedings of ION GPS'99*, Nashville, Tennessee, 14-17 September, pp. 1579-1588.

Kim, D. and R. B. Langley (1999b). "A search space optimization technique for improving ambiguity resolution and computational efficiency." Presented at GPS99, *International Symposium on GPS: Application to Earth Sciences and Interaction with Other Space Geodetic Techniques*, Tsukuba, Japan, 18-22 October; accepted for publication in *Earth, Planets and Space*.

Kim, D. and R. B. Langley (2000). "Kalman-filter-based GPS ambiguity resolution for real-time long-baseline kinematic applications." Presented at the workshop of the Central European Initiative working group on Satellite Navigation Systems, Olsztyn, Poland, 3-5 July.

Langley, R. B. (1997). "GPS receiver system noise." *GPS World*, Vol. 8, No. 6, April, pp. 40-45.

Talbot, N. (1988). "Optimal weighting of GPS carrier phase observations based on the signal-to-noise ratio." *Proceedings of the International Symposia on Global Positioning Systems*, Queensland, Australia, 17-19 October, pp. V.4.1-V.4.17.

Teunissen, P. J. G. (1990a). "An integrity and quality control procedure for use in multi sensor integration." *Proceedings of ION GPS-90*, Colorado Springs, Colorado, 19-21 September, pp. 513-522.

Teunissen, P. J. G. (1990b). "Some aspects of real-time model validation techniques for use in integrated systems." *Proceedings of KIS'90*, Banff, Canada, 10-13 September, pp. 191-200.

Teunissen, P. J. G. (1997). "The geometry-free GPS ambiguity search space with a weighted ionosphere." *Journal of Geodesy*, Vol. 71, No. 6, May, pp. 370-383.

Tiberius, C., N. Jonkman and F. Kenselaar (1999). "The stochastics of GPS observables." *GPS World*, Vol. 10, No. 2, February, pp. 49-54.

Wang, J. (1999). "Stochastic modeling for real-time kinematic GPS/GLONASS positioning." *Navigation: Journal of the Institute of Navigation*, Vol. 46, No. 4, Winter, pp. 297-305.