

# Performance of Long-Baseline Real-Time Kinematic Applications by Improving Tropospheric Delay Modeling

Don Kim<sup>1</sup>, Sunil Bisnath<sup>2</sup>, Richard B. Langley<sup>1</sup> and Peter Dare<sup>1</sup>

<sup>1</sup>Geodetic Research Laboratory, Department of Geodesy and Geomatics Engineering  
University of New Brunswick, Fredericton, New Brunswick, Canada

<sup>2</sup>Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts, USA

## BIOGRAPHY

Don Kim is a research associate in the Department of Geodesy and Geomatics Engineering at the University of New Brunswick (UNB). He has a B.Sc., M.Sc. and Ph.D. in geomatics from Seoul National University. He has been involved in GPS research since 1991 and active in the development of an ultrahigh-performance RTK system. He received the Dr. Samuel M. Burka Award for 2003 from the Institute of Navigation.

Sunil Bisnath is a geodesist at the Harvard-Smithsonian Center for Astrophysics in Cambridge, Massachusetts, where his responsibilities include management of the BARGEN geodetic network. Previously, Dr. Bisnath worked as a GPS research scientist at the Hydrographic Science Research Center at the University of Southern Mississippi. He received an Honours B.Sc. in 1993 and M.Sc. in 1995 in surveying science from the University of Toronto and a Ph.D. in geodesy and geomatics engineering from UNB in 2004.

Richard Langley is a professor in the Department of Geodesy and Geomatics Engineering at UNB, where he has been teaching since 1981. He has a B.Sc. in applied physics from the University of Waterloo and a Ph.D. in experimental space science from York University, Toronto. Prof. Langley has been active in the development of GPS error models since the early 1980s and is a contributing editor and columnist for GPS World magazine. He is a fellow of the ION and shared the ION 2003 Burka Award with Don Kim.

Peter Dare is the Chair of the Department of Geodesy and Geomatics Engineering at UNB. He obtained a B.Sc. (Hons) in land surveying sciences from North East London Polytechnic in 1980, an M.A.Sc. in civil

engineering from the University of Toronto in 1983 and a Ph.D. in geodesy from the University of East London in 1996. He joined UNB in 2000 and became the Chair of the Department in 2002.

## ABSTRACT

The University of New Brunswick and the University of Southern Mississippi are carrying out a long-term experiment in precise GPS positioning over long distances in a marine environment. The primary goal of the study, over the course of one year of data collection from daily marine ferry runs, realizing that the differential troposphere is a major limiting factor in marine positioning, is to attempt to advance positioning results by means of improved differential tropospheric modeling.

The most common approach for achieving high accuracies with GPS technology is real-time kinematic (RTK) positioning. We considered two basic requirements for long-baseline RTK. Firstly, although we process the ferry data in post-processing mode at this stage, our new approach is based on real-time data processing scenarios for actual implementation in the future. Secondly, the new approach provides positioning solutions using fixed ambiguities rather than ionosphere-free float ambiguities.

Initial RTK data processing results are presented illustrating positioning accuracy versus baseline length. And results from tests using two different tropospheric delay models are presented.

## INTRODUCTION

The University of New Brunswick (UNB) and the University of Southern Mississippi (USM) have

collaborated to devise and carry out a long-term experiment in precise GPS positioning over long distances in a marine environment. A pair of GPS reference stations (NovAtel's DL-4 receivers and GPS-600 antennas) has been deployed on either side of the Bay of Fundy in eastern Canada (see Figure 1), at the terminals of an approximately 74 km ferry route. A geodetic receiver (NovAtel's DL-4 receiver and GPS-600 antenna) has been installed on the ferry – the *Princess of Acadia*. Surface meteorological equipment has also been collocated with the three receivers. This ferry repeats the same routes between two and four times daily, depending upon the season. The Bay of Fundy is located in a temperate climate with significant seasonal tropospheric

variations (e.g., temperatures between  $-30^{\circ}\text{C}$  and  $+30^{\circ}\text{C}$ ).

The primary goal of the study is to advance the science of modeling microwave tropospheric delay over marine areas, and to test, apply, and demonstrate these advances to obtain higher accuracy (centimeter-level) positions at greater distances (10s to 100s of kilometers) from differential reference stations than is now possible, using the Global Positioning System in a post-processed (but based on real-time scenarios) fixed-ambiguity carrier-phase differential mode.

In this paper, we concentrate on initial results from the *Princess of Acadia* ferry project [Santos et al., 2004].

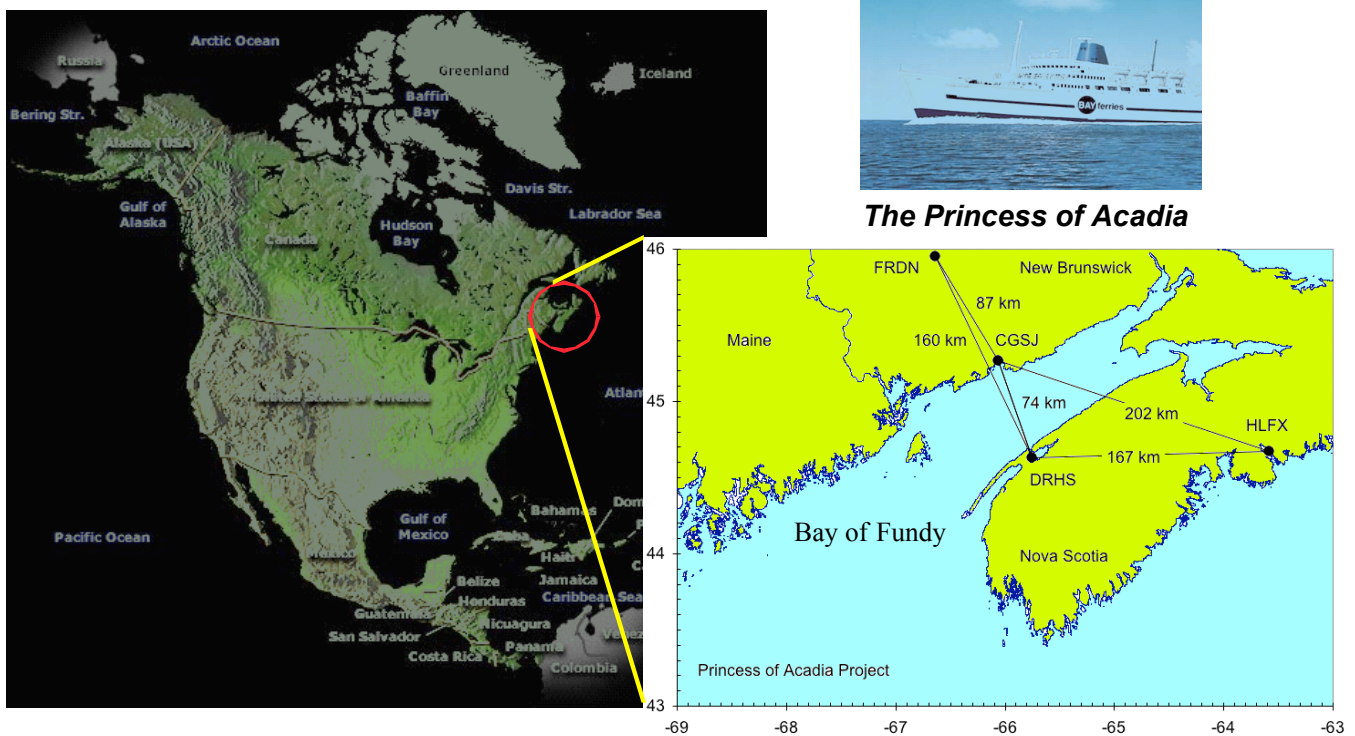


Figure 1. Local of ferry experiment: Base stations CGSJ and DRHS on either side of Bay of Fundy – ~74 km crossing.

### CONSIDERATIONS FOR A NEW APPROACH

The most common approach for achieving high accuracies with GPS technology is real-time kinematic (RTK) processing. On designing an appropriate approach for long-baseline RTK, we consider two basic requirements. Firstly, although we process the ferry data in post-processing mode at this stage, our new approach should be based on real-time data processing scenarios for actual implementation in the future. More specifically, we mean single-epoch ambiguity resolution by the real-time data processing scenarios. Secondly, unlike the previous

approach used for the ferry data processing [Bisnath et al., 2004], the new approach should provide positioning solutions using fixed ambiguities rather than the ionosphere-free float ambiguities.

One of the tools we use to assess the success of an RTK tropospheric model is the comparison between short baseline (e.g., less than 10-30 km) RTK solutions (for which RTK is generally regarded as reliable and uncontaminated by differential tropospheric uncertainties), and simultaneous position solutions from

longer RTK baselines over which the tropospheric models are being assessed.

Previous study using the ionosphere-free float ambiguities [Bisnath et al., 2004] produced sub-decimeter solution differences between the long and short baseline position estimates. In this paper, we proceed further by attempting to obtain few centimeter positioning by fixing ambiguities.

Unfortunately, many RTK systems suffer from reliability problems. These problems include a decrease in system availability with fewer satellites at mid-latitudes ( $\sim 45^\circ$  in this case) or high-latitudes, susceptibility to biases and errors such as multipath signal interference, ionospheric refraction and tropospheric refraction.

To improve RTK reliability, we use independent ambiguity resolution for the widelane, L1 and L2 observations. As the independently estimated ambiguities must satisfy a constraint (that is,  $N_w = N_1 - N_2$ ), we will have an increase in the reliability of the RTK system. This approach has been successfully tested in a gantry crane auto-steering system based on RTK [Kim et al., 2002]. Perhaps this approach may not be the best for long-baseline applications in terms of system availability. Furthermore, this approach may not take advantage of dual-frequency in cancelling the differential ionospheric delay because it will not allow us to use the ionosphere-free linear combination in estimating carrier-phase ambiguities. We are fully aware of this pitfall in the present approach and in the future we will develop an algorithm to nullify the differential ionospheric delay in the ambiguity search process without introducing the ionosphere-free linear combination.

## **BIASES AND ERRORS OF INTEREST**

There are effective mitigation strategies for all sources of RTK uncertainty, except tropospheric delay. Clock errors are eliminated by double-differencing the GPS range measurements. Ionospheric delay uncertainty is almost completely eliminated by two-frequency estimation. As mentioned above, however, this is not the case in our present approach. We will be able to do it through a nullification algorithm in the near future. GPS satellite orbit errors can be eliminated by post-processing with precise ephemerides (and have little effect for baselines up to a few 100 kilometers). Multipath uncertainties can be reduced by using special equipment: choke-ring and other multipath-resistant antennas, and receivers with multipath-estimating tracking loops. Either in real-time or post-processing applications, we can further reduce the effects of multipath through an optimal linear combination of the L1 and L2 carrier-phase observations [Kim and Langley, 2003].

Tropospheric delay is usually estimated based on either surface pressure, temperature and relative humidity measurements (at the GPS receivers being used) and/or model atmospheric predictions. This approach often inadequately accounts for horizontal and vertical spatial variations in atmospheric conditions, in particular the vertical profile of water vapour. Tropospheric delay is of greatest concern for marine vertical positioning for three reasons: (1) Tropospheric uncertainties map primarily into vertical position uncertainties. (2) Tropospheric conditions are less densely sampled at sea than over land. (3) Tropospheric uncertainties contaminate the cycle ambiguity resolution process, making longer range RTK positioning unreliable or impossible.

Much work is being done on advancing the modeling of tropospheric delay over continental areas for GPS applications in land and air transportation, and precision agriculture. For example, a network of 16 differential GPS base stations spaced 50 km apart on a 200 x 200 km grid has been established for just this purpose [Zhang and Lachapelle, 2001]. Establishing such an infrastructure at sea would be much more difficult and expensive, if not impossible.

Less RTK tropospheric delay modeling research is being done for marine applications. The marine climate and tropospheric conditions are quite distinct from those over land. Also the marine climate differs widely between temperate and tropical areas, leading to wide differences in the temporal and spatial variability of microwave tropospheric delays. One of the goals of our research is to address the need for better GPS tropospheric uncertainty modeling at sea in order to achieve longer ranges for reliable RTK vertical positioning.

## **IMPROVED TROPOSPHERIC MODELING**

The differential troposphere experienced by combining GPS measurements from a coastal base station and a near-shore reference station can differ significantly from land-based baselines. Weather fronts, temperature inversions, and other dynamic coastal weather phenomena degrade the effectiveness of present generic tropospheric delay models [Gregorious and Blewitt, 1998] to the extent that their inability to describe the behavior of the differential troposphere hampers and eventually prevents the successful ambiguity resolution process (which is required in order to obtain cm-level positions) as baselines are lengthened. As the primary limiting factor in successful long-baseline RTK (between 20 and 200 km), we propose to improve upon existing tropospheric delay models, and integrate these enhancements in RTK software signal processing.

## NOAA Experimental Tropospheric Product

NOAA has been developing a U.S. nationwide troposphere delay product. This tropospheric product is based on available weather information and estimated tropospheric delay from a GPS network. Input parameters are user location and time. Output values are wet and hydrostatic (“dry”) tropospheric delay. Fig. 2 illustrates zenith wet delay at 1:00 UTC on 24 May 2004.

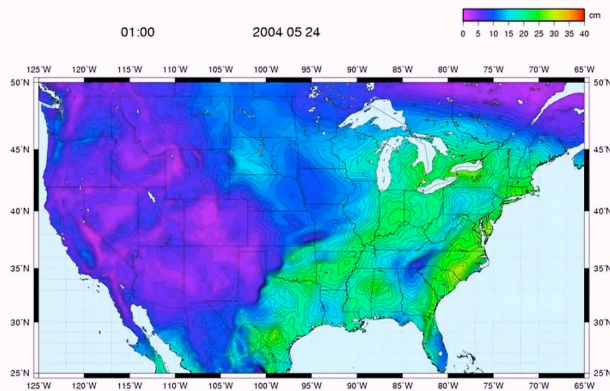


Figure 2. NOAA zenith wet delay at 1:00 UTC on 24 May 2004.

## UNB3 Tropospheric Model

The original definition of the UNB3 composite model is based on the zenith delay algorithms of Saastamoinen (1973), the mapping functions of Niell (1996), and a table of surface atmospheric values derived from the U.S. 1966 Standard Atmosphere Supplements. The kernel of the UNB3 model is a look-up table of five values of atmospheric parameters that vary with respect to latitude and day-of-year. Linear interpolation is applied between latitudes, and a sinusoidal function of the day-of-year attempts to model the seasonal variation. The parameters are total pressure, temperature and water vapour pressure at mean-sea-level, and two lapse rate parameters for temperature and water vapour. The lapse rates are used to scale the pressures and temperature to the user’s altitude. Fig. 3 illustrates mean zenith delay error of the UNB3 tropospheric model from 1992 radiosonde data [Collins and Langley, 1999].

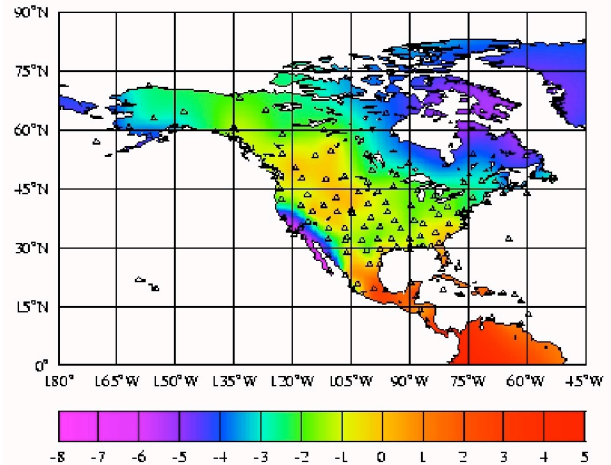


Figure 3. Mean zenith delay error [cm] of the UNB3 tropospheric model from 1992 radiosonde data.

## DATA PROCESSING STRATEGY

Using the UNB RTK software Kim and Langley [2003] initially developed for a gantry crane, auto-steering system operating under short-baseline situations and slightly modified recently for long-baseline applications, we processed data sets recorded at a 1 Hz data rate at a pair of base stations (CGSJ and DRHS) and the ferry boat on 24 May 2004.

The top panel of Fig. 4 illustrates the number of satellites tracked at the ferry boat using a  $0^\circ$  elevation mask angle. Even though the daily average number of satellites tracked ranged from 8 to 9, there were short periods where as few as only 5 or even 4 satellites were tracked. This low number of satellites results in little or no redundancy in the data processing, causing poor solutions or ambiguity processing re-initializations. Such temporary “constellation deficiencies” at mid-latitudes ( $\sim 45^\circ$  in this case) illustrates the need for little or no elevation angle masking of GPS measurements for marine applications. Applications of a larger cut-off angle would result in the reduction of noisy measurements, but would further reduce measurement strength, producing degraded position solutions or even worse – no solutions. This situation, to a certain extent, can be resolved by reducing or eliminating the elevation mask. The associated caveat though is that low elevation measurement bias arising mainly from atmospheric effects must be taken into account.

We are not reluctant in admitting the poor performance of the long-baseline RTK at this stage, especially when we attempt to fix ambiguities using single-epoch observations over long distances. Nevertheless, we used exactly the same data processing approach (but through post-processing the data) to highlight substantial issues

associated with the long-baseline RTK. To guarantee the fixed ambiguities over long distances, we decided on the following data processing strategy (refer to Fig. 4):

- Fragmentize data to validate long-baseline fixed solutions using short-baseline fixed solutions.
- Fix ambiguities on short baselines.
- Keep fixed solutions for long baselines (i.e., to the other port).

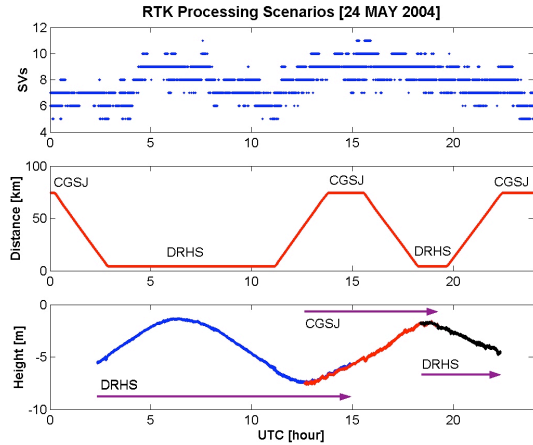


Figure 4. RTK processing scenarios. The middle panel illustrates where the ferry is located. The bottom panel shows three fragments to process and the reference stations used for RTK processing are indicated on each fragment.

## EVALUATION OBSERVABLES

One option for evaluating the tropospheric delay model is the use of the so-called ionosphere-free linear combination of carrier-phase observations:

$$\begin{aligned} \Phi_{IF} = & \rho + T + \alpha_2 \lambda_1 N_1 - \alpha_1 \lambda_2 N_2 \\ & + \alpha_2 m_1 - \alpha_1 m_2 + \alpha_2 \varepsilon_1 - \alpha_1 \varepsilon_2, \end{aligned} \quad (1)$$

where  $\rho$  is the geometric range from receiver to the GPS satellite;  $T$  is the delay due to the troposphere;  $\lambda_i$  is the carrier wavelength;  $N_i$  is the number of cycles by which the initial phases are undetermined;  $\alpha_1 \approx 1.546$  and  $\alpha_2 \approx 2.546$ ;  $m_i$  represent the effect of multipath on the carrier phases; and  $\varepsilon_i$  represent the effects of receiver noise on the carrier phases. Satellite and receiver hardware delays and other small effects have been ignored as they have negligible effect on data preprocessing. The combination observable almost completely eliminates the ionospheric delay; leaves the tropospheric delay unchanged; transforms the ambiguities into the real number domain; and magnifies the phase multipath and receiver noise.

Use of the ionosphere-free observable allows for the estimation of the tropospheric delay when the ambiguities ( $N_1$  and  $N_2$ ) and the geometric range are known. The tropospheric delay observables are given by

$$\hat{T} = \Phi_{IF} - \hat{\rho} - \alpha_2 \lambda_1 \hat{N}_1 + \alpha_1 \lambda_2 \hat{N}_2. \quad (2)$$

## INITIAL RESULTS

Since consistent ambiguity fixing was not possible using older closed-form tropospheric prediction models for processed data sets, we ignored test results from those models. Instead, we have concentrated on the comparisons between the NOAA experimental product and the UNB3 model in this paper. We have compared several aspects of the two tropospheric models including hydrostatic and wet zenith delays, performance in range and position domain, and short- and long-baseline RTK solutions.

Figures 5a to 5c show hydrostatic (“dry”) and wet components of the zenith delay predictions between two tropospheric models at the reference stations and the ferry. Over the test period, differences in hydrostatic zenith delays were a few millimeters. On the other hand, there were relatively significant differences up to several centimeters in wet zenith delays. Since the UNB3 tropospheric model provides essentially a constant prediction value of hydrostatic and wet zenith delay for each day-of-year as illustrated in Figures 5a and 5b, the effectiveness of the UNB3 model may be degraded to some extent by weather fronts, temperature inversions, and other dynamic coastal weather phenomena.

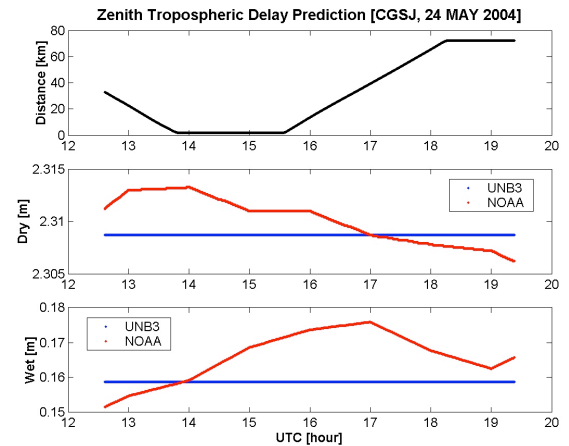


Figure 5a. Comparison of hydrostatic (“dry”) and wet zenith delay predictions between the NOAA experimental product and the UNB3 model, computed at the CGSJ reference station.



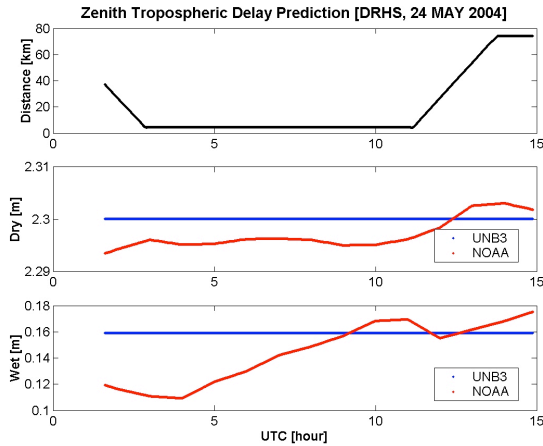


Figure 5b. Comparison of hydrostatic (“dry”) and wet zenith delay predictions between the NOAA experimental product and the UNB3 model, computed at the DRHS reference station.

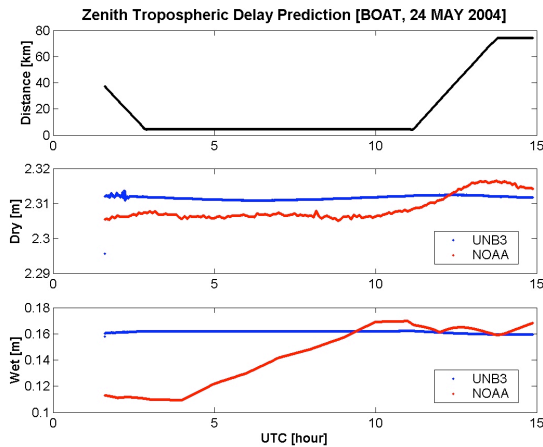


Figure 5c. Comparison of hydrostatic (“dry”) and wet zenith delay predictions between the NOAA experimental product and the UNB3 model, computed according to the ferry boat trajectory.

Compared with tropospheric delay observables driven by Eq. (2), both the NOAA experimental product and UNB3 models performed very well in the range domain for processed data sets as illustrated in Figures 6a and 6b. The residuals in the bottom panel reveal the significance of multipath and receiver system noise. If tropospheric modeling bias exists, we can also see it in the residuals. Figures 6a and 6b confirm that there is no significant tropospheric modeling bias, if any, in this case.

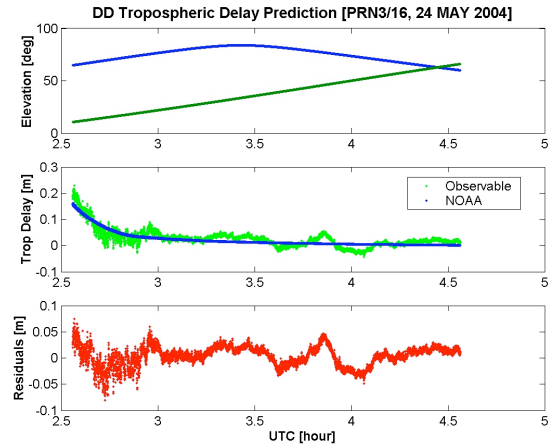


Figure 6a. Performance of the NOAA experimental product, compared with tropospheric delay observables.

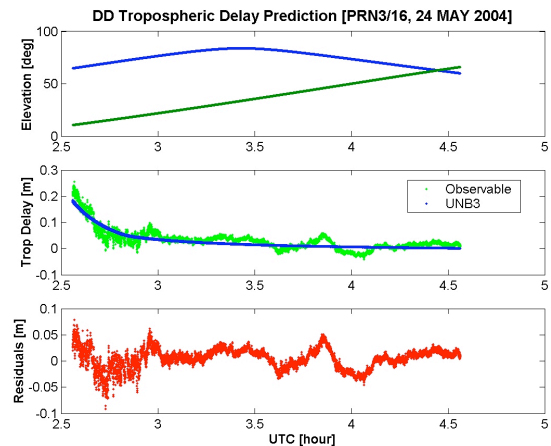


Figure 6b. Performance of the UNB3 model, compared with tropospheric delay observables.

Comparison in the position domain is illustrated in Fig. 7. To remove the effects of the differential ionospheric delays in the solutions, we used the ionosphere-free linear combination after fixing ambiguities on L1 and L2. Therefore, the differences as shown in Fig. 7 reflect directly the effects of different tropospheric models. As we expected, there was no significant changes in latitude and longitude. Most of the effects of different tropospheric models transferred into the height component.

One critical issue to be pointed out in Fig 7 is that residual zenith tropospheric delays can change some part of RTK processing scenarios and eventually may provide different positioning solutions. The spikes around 13:00 UTC show typical examples of different positioning solutions caused by the different residual zenith tropospheric delays between the NOAA experimental product and the UNB3 model.

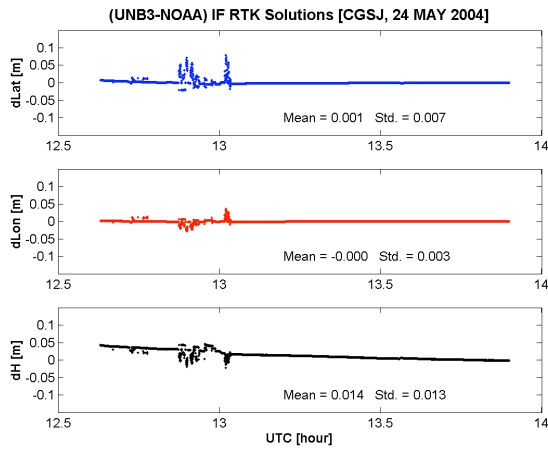


Figure 7. Comparison of ionosphere-free RTK solutions between the UNB3 and NOAA tropospheric models, corrected by the fixed L1 and L2 ambiguities.

Another option to validate the performance of the tropospheric models is the comparisons between short- and long-baseline RTK positioning solutions as illustrated in Fig. 8a and 8b. To remove the effects of the differential ionospheric delays in the solutions, we used ionosphere-free observations corrected by the fixed L1 and L2 ambiguities.

The long-baseline RTK solutions agreed very well with those of the short-baseline RTK. Differences in the horizontal and vertical solutions were less than five centimeters for the processed data sets. We outline the summary of solutions between two models in Table 1.

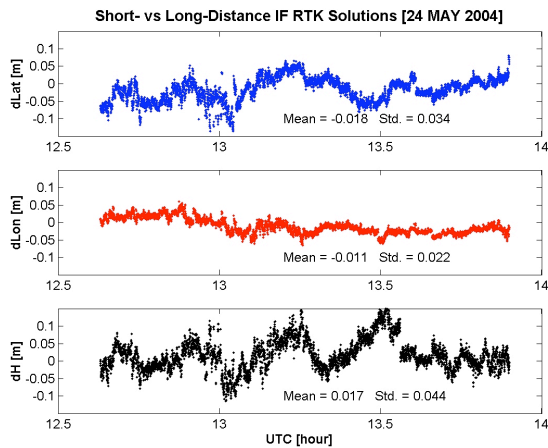


Figure 8a. Comparison of ionosphere-free RTK solutions between short- and long-baseline, applied for the UNB3 tropospheric models and corrected by the fixed L1 and L2 ambiguities.

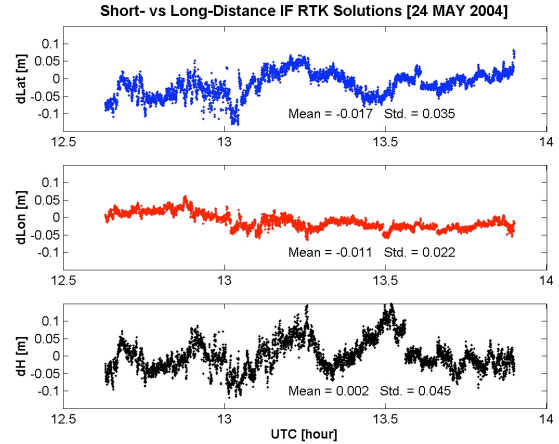


Figure 8b. Comparison of ionosphere-free RTK solutions between short- and long-baseline, applied for the NOAA tropospheric models and corrected by the fixed L1 and L2 ambiguities.

Table 1. Summary of the ionosphere-free RTK solutions between short- and long-baseline [cm]

|      | UNB3  |      |      | NOAA  |      |      |
|------|-------|------|------|-------|------|------|
|      | mean  | std  | rmse | mean  | std  | rmse |
| dLat | -1.80 | 3.40 | 3.85 | -1.70 | 3.50 | 3.89 |
| dLon | -1.10 | 2.20 | 2.46 | -1.10 | 2.20 | 2.46 |
| dH   | 1.70  | 4.40 | 4.72 | 0.20  | 4.50 | 4.50 |

Although we took a relatively reliable approach to fix ambiguities over long-distances, we experienced difficulties to some extent in obtaining RTK solutions. Some factors tend to degrade RTK performance. A typical example is illustrated in Figures 9a and 9b. As shown in the middle and bottom panels, the number of satellites used for RTK solutions decreases and HRDOP (Horizontal Relative DOP) and VRDOP (Vertical Relative DOP) increase as the ferry crosses the bay. The reason behind this occurrence is unmodelled biases, especially differential ionospheric delay. Since our current approach does not attempt to remove the differential ionospheric delay in the “ambiguity search process”, some of the observations may be screened out in the quality control routines. As the distance between the reference station and the ferry gets longer, the differential ionospheric delay tends to diverge. This divergence may introduce errors in the observables. Well designed quality control routines such as ours can detect growing errors easily and eventually remove the contaminated observations in RTK processing.

Theoretically, such a divergence in the differential ionospheric delay can be nullified in an ambiguity search process. For example, we can attempt to combine the two independent L1 and L2 ambiguity search processes into one simultaneous ambiguity search process. When a pair

of L1 and L2 ambiguity candidates is selected in the simultaneous ambiguity search process, we can completely remove the first-order differential ionospheric delays. We have not incorporated this approach yet. In the near future we will investigate the feasibility of this novel approach.

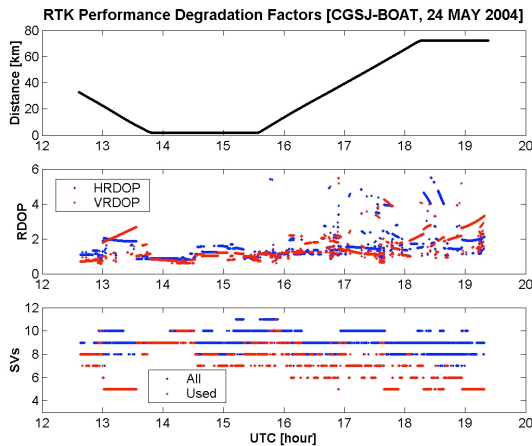


Figure 9a. RTK performance degradation in processing CGSJ and BOAT data.

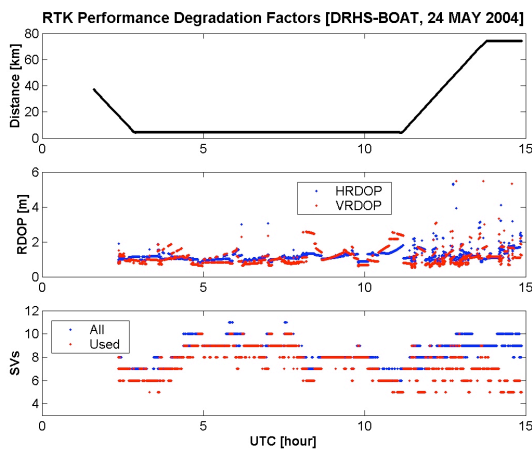


Figure 9b. RTK performance degradation in processing DRHS and BOAT data.

## CONCLUSIONS AND FUTURE WORK

Since consistent ambiguity fixing was not possible using older closed-form tropospheric prediction models for processed data sets, we ignored test results from those models. Instead, we have concentrated on the comparisons between the NOAA experimental product and the UNB3 model in this paper. We have compared two tropospheric models in several aspects including hydrostatic and wet zenith delays, performance in range and position domain, and short- and long-baseline RTK solution.

For the processed data sets on 24 May 2004, the differences in hydrostatic zenith delays were within a few millimeters. On the other hand, there were relatively significant differences up to several centimeters in wet zenith delays. Compared with tropospheric delay observables, both models performed very well in range domain. The long-baseline RTK solutions agreed very well with those of the short-baseline RTK. Differences in the horizontal and vertical solutions were less than five centimeters.

Overall performance of the two tropospheric models seemed to be quite similar for the processed data sets. However, we expect that the effectiveness of the UNB3 model may be degraded to some extent by weather fronts, temperature inversions, and other dynamic coastal weather phenomena. As a result, the UNB3 model may not be able to describe adequately the behavior of the differential troposphere. Eventually, this limitation may prevent the successful ambiguity resolution process as baselines are lengthened. However, it turns out to be true that the UNB3 tropospheric model may be a good alternative approach, especially for real-time applications as the model is provided with very simple coefficient tables and interpolation functions.

Although we took a relatively reliable approach to fix ambiguities over long-distances, we experienced difficulties to some extent in obtaining RTK solutions. Some factors tended to degrade RTK performance such as:

- A decrease of the number of satellites used for RTK solutions and an increase of HRDOP and VRDOP as the ferry crosses the bay. The reason behind this occurrence is unmodelled biases, especially differential ionospheric delay. As the distance between the reference station and the ferry gets longer, the differential ionospheric delay tends to diverge.
- Residual zenith tropospheric delays can change some part of RTK processing scenarios and eventually may provide different positioning solutions.
- Although not discussed here, the GPS system set-up for ferry data collection seemed to introduce significant multipath-like signal interference in the carrier-phase observations. This could also degrade RTK performance.

Further investigations should be carried out in the near future in the following subjects:

- Validation of new troposphere prediction models to reduce residual zenith tropospheric delay.



- Development of nullification algorithms to remove ionospheric delay in the ambiguity search process.
- Mitigation of the effects of multipath in the observations.

## ACKNOWLEDGEMENTS

The authors would like to thank the U.S. Office of Naval Research for funding of this research. Thanks also goes to the Canadian Coast Guard, Marine Atlantic (and crew of the Princess of Acadia), and Digby Regional High School for hosting GPS / meteorological stations.

## REFERENCES

- Bisnath, S., D. Wells, M. Santos, and K. Cove (2004). "Initial results from a long baseline, kinematic, differential GPS carrier phase experiment in a marine environment." Presented at IEEE PLANS 2004, Monterey, California, 26-29 April.
- Collins, J.P. and R.B. Langley (1999). "Nominal and extreme error performance of the UNB3 tropospheric delay model." Department of Geodesy and Geomatics Engineering Technical Report No. 204, University of New Brunswick, Fredericton, New Brunswick, Canada, 173 pp.
- Gregorious, T. and G. Blewitt (1998). "The effect of weather fronts on GPS measurements." *GPS World*, Vol. 9, No. 5, pp. 53-60.
- Kim, D., R.B. Langley, and S. Kim (2002). "High-precision crane guidance: Shipyard giants." *GPS World*, Vol. 13, No. 9, September, pp. 28-34.
- Kim, D. and R.B. Langley (2003). "On ultrahigh-precision positioning and navigation." *Navigation: Journal of the Institute of Navigation*, Vol. 50, No. 2, Summer, pp. 103-116.
- Niell, A.E. (1996) "Global mapping functions for the atmosphere delay at radio wavelengths." *Journal of Geophysical Research*, Vol. 101, No. B2, 3227-3246.
- Saastamoinen, J. (1973) "Contributions to the theory of atmospheric refraction." In three parts. *Bulletin Géodésique*, No. 105, 279-298; No. 106, 383-397; No. 107, 13-34.
- Santos, M., D. Wells, K. Cove, S. Bisnath (2004). "The Princess of Acadia GPS project: description and scientific challenges." Canadian Hydrographic Conference 2004, 25-27 May, Ottawa.
- Zhang, J., and G. Lachapelle (2001). "Precise estimation of residual tropospheric delays using a regional GPS network for RTK applications." *Journal of Geodesy*, Vol. 75, pp. 255-266.