PRECISE ORBIT DETERMINATION
OF THE CHAMP SATELLITE
WITH STAND-ALONE GPS

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27-30 May 2002
Copenhagen, Denmark
OVERVIEW

- Introduction of single-receiver processing technique.
- Description of UNB technique:
  - Principle.
  - Modelling.
- Spaceborne data processing:
  - Measurement description.
  - Results.
  - Analysis.
- Conclusions and further research.
SINGLE RECEIVER PROCESSING

- Purpose of research:
  - To develop processing strategy to provide precise positioning of any platform using a single, geodetic-grade GPS receiver.

- Rationale for capability:
  - Rests in desire to remove requirement of reference receiver(s) for high-precision positioning.

- Processing technique possible because:
  - GPS can provide direct, continuous, accurate, 3-dimensional positioning.
  - High-quality GPS data products (orbits and clocks) available.
  - Removal of Selective Availability (SA).
  - Have developed GPS-only technique – make no assumptions regarding platform dynamics – hence platform-independent.
PROCESSING STRATEGY (1)

- Purely geometrical – state-space, GPS-based solution.
- Simultaneously:
  - Utilize code data to compute mobile receiver position,
  - and carrier data to compute mobile receiver position change,
  - in a kinematic, sequential, least-squares filter/smooother.
- Inputs (all readily available aside from mobile receiver data):
  - Dual-frequency code and carrier measurements from mobile receiver (preferably high-rate).
  - Precise GPS constellation ephemerides.
  - Precise GPS constellation satellite clock offsets from GPST.
- Interpolate GPS-determined positions for mobile receiver state throughout trajectory.
PROCESSING STRATEGY (2)

- No need for:
  - Reference receiver or reference receiver network – that information basically provided by IGS data *products*.
  - Assumed dynamic models – time-differenced, carrier-phase observations precisely measure motion.

- Entitled:
  - Carrier-phase-connected, pseudorange point positioning.
CARRIER-PHASE-CONNECTED, PSEUDORANGE POINT POSITIONING

<table>
<thead>
<tr>
<th>Measurement</th>
<th>low precision (10s of cm), unambiguous pseudoranges</th>
<th>high precision (sub-mm), ambiguous carrier phases</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS measurement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement constituents</td>
<td></td>
<td></td>
</tr>
<tr>
<td>which must be estimated or eliminated</td>
<td>Satellite position error ××</td>
<td>Satellite position error ××</td>
</tr>
<tr>
<td></td>
<td>Satellite clock ××</td>
<td>Satellite clock ××</td>
</tr>
<tr>
<td></td>
<td>Receiver clock ××</td>
<td>Receiver clock ××</td>
</tr>
<tr>
<td></td>
<td>Troposphere ××</td>
<td>Troposphere ××</td>
</tr>
<tr>
<td></td>
<td>Ionosphere ××</td>
<td>Ionosphere ××</td>
</tr>
<tr>
<td></td>
<td>Multipath ×</td>
<td>Multipath</td>
</tr>
<tr>
<td></td>
<td>Receiver noise ×</td>
<td>Receiver noise</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>Carrier-phase ambiguity ××</td>
</tr>
</tbody>
</table>

× partially addressed
×× fully addressed

decimetre accuracy
FLOWCHART OF DATA PROCESSING

IGS GPS precise ephemerides and clock offsets → data preprocessing → filter / smooth data → interpolate between positions → platform state and state covariance → platform trajectory

platform GPS receiver data
SCHEMATIC OF FILTER PRINCIPLE

- code and carrier measurements
- initial state and covariance estimate from code

1. Compute functional and stochastic models
2. Estimate filtered state
3. State covariance estimate
4. State estimate

Sequential processing
FILTER/SMOOTHER EQUATIONS (1)

Linearized observation equations:

\[
\begin{bmatrix}
P_t - P^0_t \\
\delta\Phi_t - \delta\Phi^0_t
\end{bmatrix} =
\begin{bmatrix}
0 & A_t \\
-A_{t-1} & A_t
\end{bmatrix}
\begin{bmatrix}
\delta x_{t-1} \\
\delta x_t
\end{bmatrix} +
\begin{bmatrix}
e_t \\
\varepsilon_{t-1,t}
\end{bmatrix} +
\begin{bmatrix}
C_P \\
C_{\delta\Phi_t}
\end{bmatrix}
\]

Least-squares solution:

\[
\begin{bmatrix}
\hat{x}_{t-1} \\
\hat{x}_t
\end{bmatrix} =
\begin{bmatrix}
x^0_{t-1} \\
x^0_t
\end{bmatrix} -
\begin{bmatrix}
A^T_{t-1} C_{\delta\Phi_t}^{-1} A_{t-1} + C_{\delta x_{t-1}}^{-1} \\
-A_t^T C_{\delta\Phi_t}^{-1} A_{t-1}
\end{bmatrix}^{-1}
\begin{bmatrix}
-A^T_{t-1} C_{\delta\Phi_t}^{-1} A_t \\
A^T_t \left( C_{P_t}^{-1} + C_{\delta\Phi_t}^{-1} \right) A_t
\end{bmatrix}
\]

\[
\times
\begin{bmatrix}
-A^T_{t-1} C_{\delta\Phi_t}^{-1} w_{\delta\Phi} \\
A^T_t C_{P_t}^{-1} w_P + A_t C_{\delta\Phi_t}^{-1} w_{\delta\Phi}
\end{bmatrix}
\]
FILTER/SMOOTHER EQUATIONS (2)

Pseudorange measurement contribution:

\[ P_t - P_t^0 = A_t \delta x_t + e_t; \quad C_{P_t} \]

Carrier-phase measurement contribution:

\[ \delta \Phi_t - \delta \Phi_t^0 = -A_{t-1} \delta x_{t-1} + A_t \delta x_t + \epsilon_{t-1,t}; \quad C_{\delta \Phi_t} \]

Optimal smoothed solution:

\[ \hat{x}_s_t = C_{f_t}^{-1} \hat{x}_{f_t} + C_{b_t}^{-1} \hat{x}_{b_t} \]
ADDITIONAL MODELLING

- Observables are ionosphere-free, undifferenced pseudorange and ionosphere-free, time-differenced carrier phase.
- Modelling considerations more acute than for relative positioning:
  - Relativistic GPS SV clock correction (due to orbital eccentricity).
  - GPS SV antenna phase-centre to centre-of-mass offset.
  - GPS SV phase wind-up (especially for large sampling intervals of differenced phase).
- Model compatibility with IGS products.
The following capabilities can be realized by placing a GPS receiver modified for spaceflight (a *spaceborne* GPS receiver) aboard a Low Earth Orbiter (LEO) spacecraft:

- **Orbit determination** (position and velocity).
  - Real-time [few-metre-level].
  - Post-processed [decimetre-level].
- **Attitude determination** [sub-degree-level].
- **Timing** [sub-microsecond-level].
- **Ranging information** between spacecraft and GPS satellites for GPS signals transiting the atmosphere (limb sounding).
BENEFITS OF GPS-EQUIPPED GEOSCIENCE SPACECRAFT

- Gravity field recovery and geoid determination.
- Radar and laser altimetry.
- Interferometric Synthetic Aperture Radar mapping.
- Other remote sensing instrumentation positioning.
- Time-tagging and synchronization of scientific sensor measurements.
- Ionospheric limb sounding.
- Tropospheric limb sounding.
- Sea-surface profiling.
SPACEBORNE DATA TESTING

- Data description:
  - Orbits of the dual-frequency data from the CHAMP satellite.
  - JPL BlackJack dual-frequency receiver, tracking up to eight SVs.
  - Zenith-mounted POD antenna on choke-ring.
  - January 5th 2002.
  - 10 second data interval.
  - No elevation angle mask, but signal-to-noise mask used.
  - IGS precise GPS satellite orbits and clock offsets.

- Processing:
  - Investigate the geometric strength of measurements.
  - Test practicality and performance of technique against high-quality CHAMP conventional dynamic orbit solutions.
CHAMP SATELLITE AND SAMPLE GROUNDTRACK

- Near-circular, polar orbit
- Altitude ≈ 450 km
- Orbital Period ≈ 90 minutes
MEASUREMENT QUALITY ANOMALIES: ISSUES

- Measurement gaps in CHAMP RINEX files.
- Poor quality measurements at low elevation angles – rapid degradation.
- Periods of unexpected behaviour in all tracked carrier-phase observables.
- Entire carrier-phase outlier arcs.
- Individual epochs of single carrier-phase measurement outliers not related to cycle-slips.
- Periods of unexpected behaviour in all tracked pseudorange observables.
- Entire pseudorange outlier arcs.
- Data gaps in IGS precise GPS clock offset estimate files.

*Given all of these measurement outliers, a great deal of pre- and post-processing must be carried out.*
MEASUREMENT QUALITY ANOMALIES: EXAMPLES

- Periods of unexpected behaviour in all tracked carrier-phase observables.
- Entire carrier-phase outlier arcs.
- Entire pseudorange outlier arcs.

[Graphs showing examples of measurement quality anomalies]
DATA AVAILABILITY

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std</th>
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<tbody>
<tr>
<td>original # SVs</td>
<td>7.6</td>
<td>1.0</td>
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<tr>
<td>edited # SVs</td>
<td>7.0</td>
<td>1.2</td>
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<tr>
<td>PDOP</td>
<td>2.6</td>
<td>1.0</td>
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PROCESSING SOLUTIONS:
CODE-ONLY, FILTERED AND SMOOTHED

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>RMS</th>
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<tbody>
<tr>
<td>Code-only</td>
<td>17.6</td>
<td>1.9</td>
</tr>
<tr>
<td>Forward filter</td>
<td>5.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Backward filter</td>
<td>4.1</td>
<td>0.5</td>
</tr>
<tr>
<td>Smoother</td>
<td>1.3</td>
<td>0.4</td>
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</table>
POST-FIT RESIDUALS

- Code Residuals (m)
- Phase Residuals (m)

<table>
<thead>
<tr>
<th>res. (cm)</th>
<th>r.m.s.</th>
<th>iono-free pseudorange</th>
<th>98.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>iono-free phase diff.</td>
<td>2.1</td>
<td></td>
<td></td>
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</table>

GPS hours of week
ORBIT SOLUTION COMPARISON:
DYNAMIC JPL AND DYNAMIC GFZ (24 HRS)

<table>
<thead>
<tr>
<th>diff. (cm)</th>
<th>max. bias</th>
<th>r.m.s.</th>
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<tbody>
<tr>
<td>radial</td>
<td>25.4</td>
<td>6.4</td>
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<tr>
<td>along</td>
<td>37.5</td>
<td>-2.1</td>
</tr>
<tr>
<td>cross</td>
<td>26.4</td>
<td>1.9</td>
</tr>
<tr>
<td>norm</td>
<td>39.5</td>
<td>14.4</td>
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</table>

Processing time: hours.
ORBIT SOLUTION COMPARISON:
GEOMETRIC UNB AND DYNAMIC JPL (24 HRS)

<table>
<thead>
<tr>
<th>diff. (cm)</th>
<th>max. bias</th>
<th>r.m.s.</th>
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</thead>
<tbody>
<tr>
<td>radial</td>
<td>119.6</td>
<td>0.9</td>
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<tr>
<td>along</td>
<td>76.3</td>
<td>1.1</td>
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<tr>
<td>cross</td>
<td>53.8</td>
<td>-6.3</td>
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<tr>
<td>norm</td>
<td>138.9</td>
<td>32.9</td>
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Processing time: minutes.
**ORBIT SOLUTION COMPARISON:**

**GEOMETRIC UNB AND DYNAMIC JPL (4 HRS)**

<table>
<thead>
<tr>
<th>diff. (cm)</th>
<th>max. bias</th>
<th>r.m.s.</th>
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</thead>
<tbody>
<tr>
<td>radial</td>
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<td>3.3</td>
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<tr>
<td>along</td>
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<td>-2.6</td>
</tr>
<tr>
<td>norm</td>
<td>63.8</td>
<td>24.3</td>
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CONCLUSIONS

- Kinematic, sequential least-squares filter described.
- Dynamics-free nature of filter allows for application to any platform.
- Near-decimetre position component r.m.s difference between UNB geometric solutions and dynamic solutions in the along-track and cross-track components.
- Comparison in radial component at the few-decimetre r.m.s. level.
- Very low processing costs – few minutes of processing time with only CHAMP GPS measurements and RINEX GPS ephemeris and clock data versus hours with conventional dynamic processing.
FURTHER RESEARCH

- Process additional CHAMP data.
- Improve stochastic models in filter.
- Add some form of residual analysis to remove remaining measurement outliers.
- Simulate real-time processing with IGS predicted GPS ephemeris and clock products.