GPS Ambiguity Resolution and Validation: Methodologies, Trends and Issues

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ABSTRACT

Resolving the GPS carrier-phase ambiguities has been a continuing challenge for sub-centimeter-level high-precision GPS positioning. Once the integer ambiguities are fixed correctly, the carrier-phase observations are conceptually turned into sub-centimeter-level high-precision range measurements making it possible to attain high-precision positioning solutions. Consequently, this topic has been a rich source of GPS-research over the last decade.

A brief review of the previous work on ambiguity resolution and validation which has been carried out by many research groups from all over the world is presented in this paper. For a general understanding of these contributions, we have classified the ambiguity resolution techniques in terms of their characteristics. Current research trends and issues in ambiguity resolution and validation are described and a bibliography of representative papers is provided.

Keywords: ambiguity resolution and validation, classification of ambiguity resolution techniques, research trends and issues in ambiguity resolution and validation.

1. INTRODUCTION

For the last decade or so, it has been a continuing challenge to resolve the GPS carrier-phase integer ambiguities. From the first demonstration of the use of the GPS carrier-phase observations for sub-centimeter-level precision positioning and surveying, to the latest development of the instantaneous ambiguity resolution on-the-fly (OTF) techniques, a lot of ambiguity resolution techniques have been proposed by many groups from all over the world. According to a recent report of the Special Study Group (SSG) 1.157 of the International Association of Geodesy (IAG), 300 of the 325 catalogued papers on ambiguity resolution and validation have been published during the last decade. Furthermore, research in this area tended to increase recently (Table 1). Why is interest in ambiguity resolution and validation growing? The answer may be found in the interesting scientific aspects of the problem such as integer least-squares estimation theory, ambiguity search algorithms, and the extensive number of scientific applications which can benefit by the highest-possible GPS positioning accuracies. Also, there are large commercial interests in the problem as well.

Table 1. Number of papers on ambiguity resolution and validation

Years	No. of papers
1981-1984	1
1985-1989	24
1990-1994	123
1995-1999	177
Total	325

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The GPS carrier-phase ambiguity represents the arbitrary counter setting (an integer value) of the carrier-phase cycle tracking register at the start of observations of a satellite (phase lock), which biases all measurements in an unbroken sequence of that satellite's carrier-phase observations. Once the integer ambiguities are fixed correctly, the carrier-phase observations are conceptually turned into millimeter-level high-precision range measurements and hence it is possible to attain sub-centimeter-level positioning solutions. However, fixing the integer ambiguities is a non-trivial problem, especially if we aim at computational efficiency and high performance (or success rate). Therefore, this topic has been a rich source of GPS-research over the last decade.

In general, algorithms for solving the integer ambiguities have been developed for two different applications. The first group of the algorithms has been developed for applications using multi-reference stations in static mode. Multi-reference stations are occupied for several hours or even several days. Inter-station distance can reach thousands of kilometres. Whereas the second group of the algorithms has been developed for rapid-static, kinematic and navigation applications. Only two stations are usually involved with at least one station moving. The maximum distance between the stations is a few tens of kilometres. Time of occupation is the order of seconds to minutes or even instantaneous. However, conceptually there are no differences between the two applications, and the research directed to one application can benefit from research conducted for the other. Moreover, particular interests such as real-time long-baseline kinematic applications integrate the two approaches.

For a general understanding of this research area, it may be useful to take a look at previous work in the field and various research projects in progress. One good source of information where we can get an overview of research trends and issues on ambiguity resolution is the IAG Web site, particularly the reports of the SSGs which belong to Section I. IAG Section I is concerned with the scientific aspects of measurements and the analysis of regional and global geodetic networks as well as satellite, inertial, kinematic and marine positioning. One of the main activities of an SSG is the international coordination of the ongoing research in its field. The following SSGs are of particular interest:

1) SSG 1.154 on "Quality issues in real time GPS positioning",
2) SSG 1.157 on "GPS ambiguity resolution and validation",
3) SSG 1.179 on "Wide area modeling for precise satellite positioning".

This paper attempts to review briefly the main issues and research trends in ambiguity resolution and validation. First, we will summarize the previous work which has been carried out by many research groups from all over the world. Then, we will take a look at recent research activity in the area.

2. AMBIGUITY RESOLUTION TECHNIQUES

Comparison of the ambiguity resolution techniques is not easy and not always feasible. The terms of reference of SSG 1.157 clarifies this point:

> "Despite the large effort spent by many groups from all over the world in devising various schemes, knowledge about their theoretical foundation, and how the schemes are related to each other, is still lacking. Different terminology is used and comparisons between methods are rare. Due to a lack of knowledge about the various methods, the implementations used in the comparisons (if made at all) are not always complete, thereby making the test results unreliable. Moreover, results reported of one particular method, are often difficult to relate to the results of another method, due to lacking knowledge of the characteristics of the data and the type of computer that was used."

As a result, comparison of the ambiguity resolution techniques in terms of computational efficiency and performance is not always reliable. To avoid this pitfall, we will restrict our interest to categorizing the techniques in terms of individual characteristics.

Classification based on measurement information utilization

So far, there is only one paper which has presented a comprehensive study of the classification of the ambiguity resolution techniques [*Hatch and Euler*, 1994]. In the paper, many ambiguity resolution techniques are classified according to how they attempt to make use of the information contained within the receiver measurements. Although some papers briefly describe (usually in an introduction) their own classification, there is no significant difference from that of *Hatch and Euler*. The classification is three-fold:

1) Ambiguity resolution in the measurement domain,
2) Search technique in the coordinate domain,
3) Search technique in the ambiguity domain.

Table 2. Characteristics of ambiguity resolution techniques.

Technique	Principal Author(s)	Ambiguity Search Method*	Data Processing Method*	Search Space Handling Method*
LSAST	Hatch	Independent	Single-epoch	None
FARA	Frei and Beutler	All	Multi-epoch	Conditional
Modified Cholesky Decomposition	Euler and Landau	All	Multi-epoch	None
LAMBDA	Teunissen	All	Multi-epoch	Transformation /Conditional
Null Space	Martin-Neira	Independent	Single-epoch	Transformation
FASF	Chen and Lachapelle	All	Multi-epoch	Conditional
OMEGA	Kim and Langley	Independent	Single/Multi-Epoch	Transformation /Conditional

* Refer to the section “Classification based on methodology” for details.

The first class is the simplest ambiguity resolution technique which uses C/A or P-code pseudoranges directly to determine the ambiguities of the corresponding carrier-phase observations. Since the precision of raw C/A or even P-code pseudoranges is, however, generally not good enough to determine the integer ambiguities, inter-frequency linear combinations of the L1 and L2 observations are used with a smoothing process for the estimated ambiguities. There are a few papers which give comprehensive studies of the inter-frequency linear combinations (see, for example, *Cocard and Geiger* [1992], *Collins* [1999]).

The second class of algorithms includes the very first ambiguity resolution technique developed, namely the Ambiguity Function Method (AFM) [*Counselman and Gourevitch*, 1981; *Remondi*, 1984]. This technique uses only the fractional value of the instantaneous carrier-phase measurement and hence the ambiguity function values are not affected by the whole-cycle change of the carrier phase or by cycle slips. Despite significant improvement of the original algorithm by *Han and Rizos* [1996], the technique provides relatively poor computational efficiency and consequently it is of little import other than of historical interest.

The third class comprises the most abundant group of techniques which are based on the theory of integer least-squares [*Teunissen*, 1993]. Parameter estimation under the theory is carried out in three steps – the float solution, the integer ambiguity estimation, and the fixed solution. Each technique makes use of the variance-covariance matrix obtained at the float solution step and employs different ambiguity search processes at the integer ambiguity estimation step. The following are some representative techniques in the class: the Least-Squares Ambiguity Search Technique (LSAST) [*Hatch*, 1990]; the Fast Ambiguity Resolution Approach (FARA) [*Frei and Beutler*, 1990]; the modified Cholesky decomposition method [*Euler and Landau*, 1992]; the Least-Squares AMBiguity Decorrelation Adjustment (LAMBDA) [*Teunissen*, 1994]; the null space method [*Martin-Neira et al*., 1995]; the Fast Ambiguity Search Filter (FASF) [*Chen and Lachapelle*, 1995]; and the Optimal Method for Estimating GPS Ambiguities (OMEGA) [*Kim and Langley*, 1999a].

Classification based on methodology

The ambiguity resolution techniques which belong to the first two classes delineated in the previous section are so straightforward that further discussion is not needed. As for the third class, however, an overarching point of view may be helpful in understanding the various techniques because each technique has more or less different characteristics. Therefore, in the remainder of this paper we will restrict our discussion to the techniques in the third class.

Before we investigate the ambiguity resolution techniques further, we will consider the general goals of the techniques and discuss both the common and disparate aspects of the techniques. As can be seen in many papers, the general goals that the ambiguity resolution techniques try to achieve are to increase the performance of the integer ambiguity estimation and to improve the computational efficiency of the ambiguity search process. The performance of the integer ambiguity estimation, which means the capability to discriminate a correct ambiguity set from all candidate sets, can be generally increased by means of more realistic stochastic models for receiver system noise (or observation noise) and well-defined validation procedures for the solutions. Unfortunately, it is not easy to distinguish one technique from the others based on their performance because almost all the techniques have common aspects as far as performance is concerned. In fact, it is the computational efficiency of the ambiguity search process rather than the performance that distinguishes the different ambiguity resolution techniques.

Another common feature of the ambiguity resolution techniques can be found in the objective functions which are derived from the integer least-squares problem. Many ambiguity resolution techniques which perform a search in the ambiguity domain are basically based on the minimization of the quadratic form of the residuals. Principally, this minimization problem is referred to as an integer least-squares problem due to the integer-constraint for the ambiguity parameters. The same objective function, which is related to the minimization of the quadratic form of the residuals under the integer least-squares problem, can be derived using different approaches [*Euler and Landau*, 1992; *Teunissen*, 1993; *Kim and Langley*, 2000a].

With respect to the computation of the objective function, there are two basic approaches: a "*single-epoch*" (or instantaneous) approach and a "*multi-epoch*" approach. A single-epoch approach uses one epoch's observations while a multi-epoch approach uses many sequential observations in computing the quadratic form of the residuals. As far as the performance of ambiguity resolution is concerned, the single-epoch approach may not attain a higher success rate than the multi-epoch approach because the single-epoch approach often fails to find correct ambiguity parameters. This is due to the difference between a local and a global minimum for the quadratic form of the residuals. When the observations are significantly contaminated by biases such as multipath, residual atmospheric effects, satellite orbit error and so on, a local minimum which is determined using one epoch's observations is apt to be biased. On the other hand, a global minimum which is determined using many sequential observations over a relatively long time span is unbiased as long as the behaviour of biases can be assumed to be a random process over the time span. Therefore, the single-epoch approach should be used with a procedure which improves ambiguity resolution performance. It has been reported that the performance of the single-epoch approach can be improved when a linear filter for the residuals is employed [*Borge and Forssell*, 1994] or when the time average of the objective function is used [*Martin-Neira et al*., 1995].

As turns out to be clear (to some degree) from the above discussion, the ambiguity resolution techniques have different characteristics in terms of the computational efficiency of the ambiguity search process and hence we can classify the techniques according to their approaches for the search process. In classifying the techniques, we kept in mind two questions: 1) How do the techniques describe or limit the ambiguity search space? and 2) How do the techniques deal with the ambiguity parameters?

The computational efficiency of the ambiguity search process can be improved in several ways. A general approach is to reduce the ambiguity search space which comprises the ambiguity candidate sets. In this case, the correct ambiguity set should be retained in the reduced search space once the original search space includes it. We can find two methods for this approach. One is the search domain transformation method which transforms the original ambiguity sets into the corresponding ones in a transformed space. The reduction effect is usually gained through a "many-to-one" relationship between the original and transformed sets, and/or through redefining a more efficient search space than original one [*Abidin*, 1993; *Martin-Neria et al.*, 1995]. The other method for reducing the search space is to define the conditional search ranges in multi-level searches (e.g., FARA and FASF). This method is based on the fact that the ambiguity parameters of lower search levels are conditioned on those of upper search levels. Some techniques use two methods simultaneously (e.g., LAMBDA and OMEGA). Another approach for improving the computational efficiency is to find a more efficient computational algorithm for the quadratic form of the residuals. There have not been many investigations into this approach except for studies on the modified Cholesky decomposition method [*Euler and Landau*, 1992] because the search space reduction methods usually obtain higher computational efficiency anyway. Moreover, we have already well-known computational algorithms for the quadratic form of the residuals, such as the singular value decomposition and the Cholesky decomposition. Note that the modified Cholesky decomposition is frequently used in the context of the ambiguity resolution techniques due to a symmetric nonnegative definite matrix in the quadratic form of the residuals. Recent studies by *Kim and Langley* [1999b, 2000a] show that computational efficiency can be significantly improved using alternative algorithms for the quadratic form of the residuals.

As mentioned previously, parameter estimation under the integer least-squares problem is carried out in three steps – the float solution, the integer ambiguity estimation, and the fixed solution. Regardless of which ambiguity resolution technique is used, the same computational algorithms can be employed for the float and fixed solutions. On the other hand, the procedures of the integer ambiguity estimation depend on the specific ambiguity search technique adopted. We can classify the ambiguity search techniques into two classes according to how they handle the ambiguity parameters in the ambiguity search process – an "*all-ambiguity-search*" method (e.g., FARA, LAMBDA, FASF and the modified Cholesky decomposition method) and an "*independent-ambiguity-search*" method (e.g., LSAST and OMEGA). In generating the ambiguity search space, the first method uses all the ambiguity parameters while the second one uses only independent ambiguity parameters which provide an unique solution for the system (or observation equations) and hence dependent ambiguity parameters are determined once the independent ones are given [*Hatch*, 1990]. Generally, the independent-ambiguity-search method has a great efficiency in generating the ambiguity search space but computational efficiency of the method is not improved as much as expected because of the computational burden of the dependent ambiguity parameters. *Kim and Langley* [1999b, 2000a] proposes alternative algorithms to overcome the problem.

3. RESEARCH TRENDS AND ISSUES

Over the past decade, the ambiguity resolution techniques have been improved with the ambiguity search process made more efficient in order to make the ambiguity resolution techniques practical. As a matter fact, it is "*computational efficiency*" that the ambiguity resolution techniques have aimed at. Unfortunately, the race for more efficient ambiguity search algorithms seems to be already saturated because current achievements in computational efficiency seem sufficient for many practical applications. For example, some techniques such as FASF, LAMBDA and OMEGA provide ambiguity and positioning solutions within a few tens of milliseconds according to their developers. Such speed is more than satisfactory for most applications.

Although many of the currently available techniques are quite efficient, this does not necessary mean that further investigations for better ambiguity search algorithms are pointless.

On the contrary, ambiguity resolution techniques have not usually been described in terms of a generalized (or standard) procedure which includes: a functional (or deterministic) model which describes the relationship between observations and unknown parameters; a stochastic model which represents the noise characteristics of the observations; a quality control scheme which handles cycle slips (or outliers); and a parameter-estimation scheme which determines ambiguity parameters as well as navigation solutions. As a result, it is not always clear if the results reported in the literature are isolated ("best") cases or can be reproduced at any place on the earth at any time during the day. In fact, we need to consider the "*reliability*" of the techniques. IAG SSG 1.154 on "Ambiguity Resolution and Validation" had investigated the problem during the years 1995-1999. Unfortunately, the lack of a generalized procedure is still present although progress has been made. Furthermore, there is no general agreement on the form of the generalized procedure up to now.

As was mentioned previously, the general goal of the ambiguity resolution techniques includes the "*performance*" (or success rate) of ambiguity resolution. Although no comprehensive study has yet been performed, it is evident from many papers that concern about this aspect of ambiguity resolution has been growing. The issues related to performance are two-fold: 1) How do we increase performance? and 2) How do we evaluate performance? The first issue is involved with the *qualitative* realization of performance, whereas the second issue is related to the *quantitative* evaluation of performance.

To obtain optimal solutions in the least-squares estimation, both a functional and a stochastic model should be specified appropriately. As long as the models are correct, in principle, the optimal solutions are not biased and hence we can obtain correct solutions. The same is true for the ambiguity parameter estimation because it is based on the integer least-squares estimation theory. Therefore, the realization of high performance (i.e., the determination of correct ambiguity parameter values) depends on how correctly we can establish the functional and the stochastic models. With respect to the function model, specific error modeling and parametric modeling for the error sources – ionospheric delay, tropospheric delay, satellite orbit error, multipath and so on – are essential concerns. Although intensive research has been conducted in this area, interest is still growing, particularly for the applications requiring real-time, long-baseline or kinematic solutions. On the other hand, stochastic modeling has received less attention than functional modeling. However, the stochastic modeling turns out to be a crucial research topic as the interest in the performance of ambiguity resolution increases. The race for more realistic stochastic models is in full swing at this moment with many recent developments: the elevation-angle dependent function approach [Jin, 1996]; the signal-to-noise ratio (SNR) or alternatively the carrier-to-noise-power-density ratio (C/N_0) approach [*Hartinger and Brunner*, 1998; *Barnes et al.*, 1998; *Collins and Langley*, 1999]; the (adaptive) least-squares estimation approach [*Wang et al.*, 1998a; *Wang*, 1999; *Tiberius and Kenselaar*, 2000] and the real-time estimation approach [*Han*, 1997; *Kim and Langley*, 2000c]. Fundamental discussions about the observation noise were given by *Langley* [1997] and *Tiberius et al*. [1999]. IAG SSG 1.179 on "Wide Area Modeling for Precise Satellite Positioning" will be closely related to the issues of functional and stochastic modeling during the years 2000-2003.

There are two approaches for the evaluation of performance: 1) the performance evaluation function approach and 2) the discrimination function approach. The first approach tries to evaluate the performance of the integer ambiguity parameters using the probabilistic properties of the integer ambiguity estimators. *Teunissen* [1998, 1999] has proposed a performance evaluation function, particularly for the integer bootstrapping (i.e., sequential conditional integer rounding) technique which was adopted in the LAMBDA method. On the other hand, the second approach tries to measure discrimination power between the best ambiguity candidate and the second-best one. Although some discrimination test procedures have been proposed during the past decade, more reliable test procedures which can quantify discrimination power have been proposed only recently [*Han*, 1997; *Wang et al.*, 1998b]. Compared with the activities for the qualitative realization mentioned previously, the research on the quantitative evaluation is less intense so far. Nevertheless this topic is ripe enough to start a race.

With an increased level of research on (real time) long-baseline (kinematic) applications recently, some of this research has shown up in the ambiguity resolution arena. Although we have already noted the extensive ambiguity resolution research over the past decade, this research was usually directed towards post-processing and static applications. But all the issues mentioned above (i.e., computational efficiency, reliability and performance) are inherent in the real-time, long-baseline, or kinematic applications. For example, functional modeling turns out to be difficult because of the decorrelation of such biases as ionospheric delay, tropospheric delay and satellite orbit error in long-baseline situations, and the quasi-random behavior of multipath in kinematic situations. As a result, stochastic modeling also turns out to be more difficult. On the other hand, those problems can be handled relatively easily in static and short-baseline applications. Conventionally, the multi-reference station approaches have been proposed to overcome the problems in (post-processing) long-baseline (static) applications (see *Fotopoulos* [2000]). The capability of real-time and kinematic processing has been investigated using the virtual reference station approach [*van der Marel*, 1998; *Odijk et al.*, 2000] and the parametric estimation approach [*Kim and Langley*, 2000b].

4. CONCLUDING REMARKS

A brief review of the research trends and issues on ambiguity resolution and validation has been presented in this paper. At first, we took a look at the work carried out by many research groups from all over the world. Since it is not always clear if the results reported in the literature are isolated (or ideal) cases or can be reproduced at any place at any time, comparison of the different ambiguity resolution techniques based on the literature is not always reliable. To avoid this pitfall, we tried to give our viewpoints in classifying the techniques solely in terms of their characteristics. This may be helpful for a general understanding of this research area.

Considering the general goals of the ambiguity resolution techniques, it is the computational efficiency of the ambiguity search process rather than the performance that characterizes the individual ambiguity resolution techniques. For at least a decade, there has been an intensive competition to develop more efficient ambiguity search algorithms because computational efficiency was essential to make the ambiguity resolution techniques practical. As a result, current achievements in computational efficiency seem sufficient for many practical applications. This trend has switched the race to the performance of ambiguity resolution. Currently, work on two issues related to the performance is in full swing: i.e., the qualitative realization of high performance and the quantitative evaluation of performance. Both issues are equally important in terms of accuracy and availability in designing a system.

We are facing a new challenge in the demands of real-time long-baseline kinematic applications. There is an attractive driving force in the challenge because all the issues related to the ambiguity resolution (i.e., computational efficiency, reliability and performance) are inherent to these applications. These applications require the integration of the knowledge and experiences that we have obtained from the conventional approaches for short-baseline and long-baseline analyses, for static and kinematic applications, and for post-processed and real-time evaluations.

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