GPS Phase-Connected, Precise Point Positioning of Low Earth Orbiters

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BIOGRAPHIES

Sunil Bisnath received an Honours B.Sc. in 1993 and an M.Sc. in 1995 in Surveying Science from the University of Toronto. For the past five years he has been a Ph.D. candidate in the Department of Geodesy and Geomatics Engineering at the University of New Brunswick. During this time he has worked on a variety of GPS-related research and development projects, the majority of which have focused on the use of GPS for space applications.

Richard Langley is a professor in the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, where he has been teaching and conducting research since 1981. He has a B.Sc. in applied physics from the University of Waterloo and a Ph.D. in experimental space science from York University, Toronto. Professor Langley has been active in the development of GPS error models since the early 1980s and is a contributing editor and columnist for GPS World magazine.

ABSTRACT

A completely geometric approach for precise orbit determination (POD) of low earth orbiter (LEO) spacecraft has been devised by the authors which does not use dynamic models, but only data from the GPS receiver onboard the LEO and the International GPS Service (IGS) precise GPS ephemeris product. The strategy relies on combining the time-continuous measurement strength of the pseudorange and carrier-phase observables. The result is a straightforward, efficient procedure, which has the striking characteristics of providing a single receiver solution and is receiver platform-independent.

This paper describes the mathematical model development of a kinematic, sequential least-squares filter/smoother implementing this form of GPS positioning. The strategy is then applied to orbit determination using data from the TOPEX/Poseidon, and CHAMP missions. The overall accuracy is assessed, with good quality data producing few-decimetre orbits in terms of total displacement error.

INTRODUCTION

Interest in POD of LEOs using GPS has been growing rapidly. Conventional GPS-based POD strategies rely on data from a network of terrestrial GPS receivers as well as the spaceborne receiver, and involve complex, lengthy estimation procedures integrating the GPS data with high-fidelity dynamic models for the LEO. These strategies rely greatly on the GPS measurement strength, especially for low altitude spacecraft.

Given the accuracy of GPS-based positioning, the availability of precise GPS data products, and the removal of Selective Availability (SA), we were interested in determining if single-receiver, GPS-only POD is possible. To reach this end, we have developed a GPS-only POD strategy – the geometric strategy. This paper describes this processing strategy, provides test results and analysis from a number of data sets from various satellites, and provides conclusions based on the current results, as well as research plans to improve processing performance.

GEOMETRIC ORBIT DETERMINATION STRATEGY

Classical OD was designed to incorporate sparse, often imprecise measurement data that are not necessarily three-dimensional in nature. The advent of GPS has allowed for the possibility of *in situ* continuous, accurate, three-dimensional position information to be collected. Also, in mission scenarios involving low altitudes and irregularly-shaped spacecraft, the GPS measurements can potentially provide more accurate position estimates than dynamics-based strategies.

Therefore a purely geometrical, GPS-based orbit determination strategy has been proposed [Bisnath and

Poster presented at GNSS 2001, 8-11 May 2001, Seville, Spain.

Langley, 1999; Bisnath and Langley, 2001], utilizing only readily-available International GPS Service (IGS) data products (see, *e.g*., Neilan *et al*. [1997]) and LEO receiver measurements. This provides for very efficient, straightforward processing and takes full advantage of the precise, three-dimensional and continuous nature of GPS measurements, as well as the existing GPS data infrastructure. The original proposal of this strategy [Bisnath and Langley, 1999] entailed the use of a network of static, terrestrial reference receivers to be used to virtually eliminate GPS satellite and receiver clock offsets in a double-differenced, relative measurement scheme. However, with the removal of Selective Availability (SA) from the GPS signal, precise GPS satellite clock information can be interpolated without fear of significant degradation and the strategy can be carried out with just the *single* spaceborne receiver.

The processing flow of the strategy is shown in Figure 1. The input pseudorange and carrier-phase data are pre-processed to detect outliers, cycle slips, *etc*. and then used to form the processing observables. The LEO position is then estimated with the filter described in the following section. By applying an accurate interpolation procedure, LEO state estimates at epochs between GPS measurement epochs can also be determined producing the final orbit.

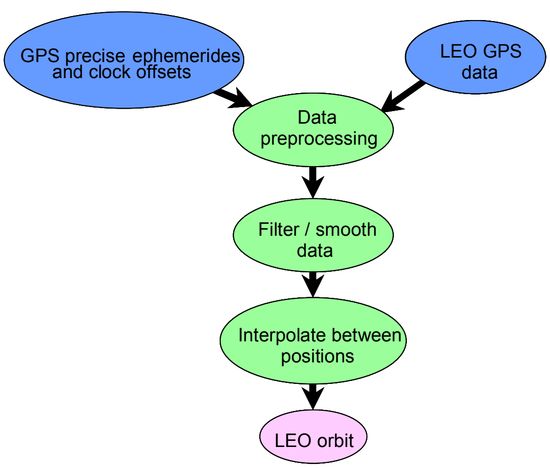

Figure 1: Processing flow of the geometric strategy.

Phase-Connected, Point Positioning Filter Design

The basis of the filter is the combination of pseudorange (code-phase) and carrier-phase measurements stemming from research originated by Hatch [1982]. The crux of carrier/pseudorange synthesis is essentially the use of averaged noisy code-phase range measurements to estimate the ambiguity term in the precise carrier-phase range measurements. The longer the pseudorange averaging, the better the carrier-phase ambiguity estimate.

The carrier/pseudorange averaging periods are typically short in spaceborne applications due to the relatively fast motion of the LEO, necessitating frequent changing of GPS satellites being tracked by the receiver. Such a situation does not allow for the highest precision of the technique to be attained. However by performing the averaging in the *position* rather than the *range* domain, previous position solutions can be used in estimating present and future position solutions. In essence, the pseudoranges provide coarse position estimates and the relative carrier phase measurements provide precise position change estimates. The position change estimates are used to map all of the position estimates to one epoch for averaging.

Similar processing filters have been described with various relative positioning formulations and inputs. These include those developed by Yunck *et al.* [1986], and Kleusberg [1986]. In fact, Yunck *et al.* proposed this type of filter in 1986 for the specific purpose of geometric GPS-based LEO orbit determination. However, this strategy was abandoned for others, since at the time a global array of terrestrial GPS reference stations did not yet exist to provide sufficiently precise GPS ephemerides.

The observables fed to the filter are the ionosphere-free, undifferenced pseudorange and the ionosphere-free, time-differenced carrier-phase. For point positioning, a number of additional modelling considerations must be taken into account above and beyond those required for relative positioning. These include the relativistic GPS satellite clock correction due to the eccentricity in the satellite orbits; GPS satellite antenna phase centre to centre of mass offset; GPS satellite phase wind-up due to the relative rotation of the satellites with respect to the receiver; sub-diurnal variations in earth rotation; and consistency between the models used in the generation of the precise GPS orbits and clocks, and those used in the point positioning processing.

Given that this phase-connected, point positioning technique does not take into account the LEO dynamics nor makes any assumptions regarding dynamics, it can therefore be applied to any platform. This fact greatly enhances the utility of the approach and was used in previous research for testing purposes [Bisnath and Langley, 2001].

Filter Models and Solution

The linearised filter observation model in matrix form is

$$\begin{bmatrix} P_t - P_t^0 \\ \delta\Phi_t - \delta\Phi_t^0 \end{bmatrix} = \begin{bmatrix} 0 & A_t \\ -A_{t-1} & A_t \end{bmatrix} \begin{bmatrix} \delta x_{t-1} \\ \delta x_t \end{bmatrix} + \begin{bmatrix} e_t \\ \varepsilon_{t-1,t} \end{bmatrix};$$

$$C_{P_t},\ C_{\delta\Phi_t}, \qquad (1)$$

where P_t and P_t^0 are the pseudorange measurement and predicted value, respectively; $\delta\Phi_t$ and $\delta\Phi_t^0$ are the time-differenced carrier phase measurement and predicted value, respectively; δx_{t-1} and δx_t are the estimated corrections to the LEO receiver position and clock at epoch t-1 and t, respectively; A_{t-1} and A_t are the measurement partial derivatives with respect to the LEO receiver position and clock estimates for epochs t-1 and t, respectively; e_t and ε_{t-1} are the measurement errors associated with P_t and $\delta\Phi_t$, respectively; and C_{P_t} and $C_{\delta\Phi_t}$ are the covariance matrices for P_t and $\delta\Phi_t$, respectively. Note that at present the pseudorange and carrier phase measurements are assumed uncorrelated between observables and between observations.

The *best* solution for (1), in a least-squares sense, is

$$\begin{bmatrix} \hat{x}_{t-1} \\ \hat{x}_t \end{bmatrix} = \begin{bmatrix} x_{t-1}^0 \\ x_t^0 \end{bmatrix} - \begin{bmatrix} A_{t-1}^T C_{\delta\Phi_t}^{-1} A_{t-1} + C_{x_{t-1}}^{-1} & -A_{t-1}^T C_{\delta\Phi_t}^{-1} A_t \\ -A_t^T C_{\delta\Phi_t}^{-1} A_{t-1} & A_t^T \left(C_{P_t}^{-1} + C_{\delta\Phi_t}^{-1} \right) A_t \end{bmatrix}^{-1} \times \begin{bmatrix} -A_{t-1}^T C_{\delta\Phi_t}^{-1} w_{\delta\Phi} \\ A_t^T C_{P_t}^{-1} w_P + A_t C_{\delta\Phi_t}^{-1} w_{\delta\Phi} \end{bmatrix}, \quad (2)$$

where $\hat{x} = x^0 + \delta x$ (the estimate is equal to the approximate initially assumed value plus the estimated correction); w_P and $w_{\delta\Phi}$ are the misclosure vectors for the pseudoranges and time-differenced carrier phases, respectively; and $C_{x_{t-1}}^{-1}$ is the LEO receiver position and clock covariance based on the last epoch's observations.

The position estimate at the previous epoch, t-1, is used to estimate the position at epoch t and so on for the moving LEO. (2) represents a kinematic, sequential least-squares filter. This type of filter is a subset of the Kalman filter. The filtering process is illustrated in Figure 2 and further details can be found in Bisnath and Langley [2001].

DATA PROCESSING RESULTS AND ANALYSIS

In order to validate the geometric strategy, a number of tests were conducted using the latest version of the developed processing software. This software is based on the University of New Brunswick's scientific GPS processing package DIPOP [Kleusberg *et al*., 1993] and its components are not yet all complete. Where necessary, mention will be made of additional processing or modelling that is required.

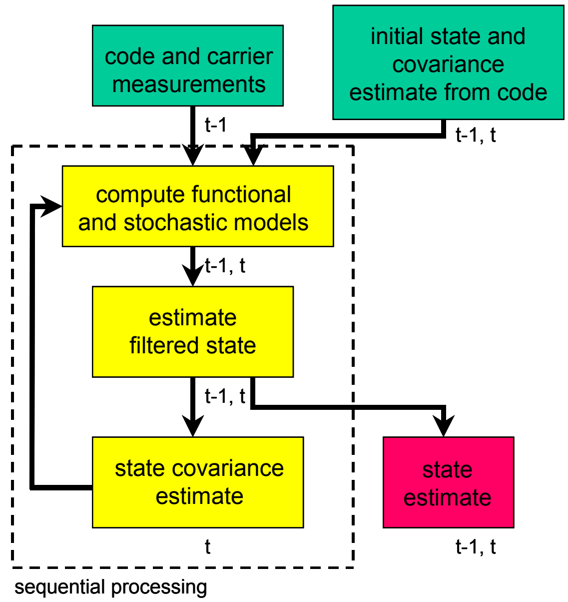

Figure 2: Combination of pseudorange and carrier-phase observations in the kinematic, sequential least-squares filter.

Topex/Poseidon Data

The one day of Topex/Poseidon data processed here were collected on 13 November 2000. A description of the pertinent orbit and receiver parameters is given in Table 1. The appropriate IGS precise GPS constellation orbit and clock offset file was the only additional input to the processing.

Parameter	Description
Launch date	10 August 1992
Nominal altitude	1335 km
Inclination	66°
Receiver type	GPSDR
No. of SVs tracked	up to 6
Provided observables	L1, P1
Provided data interval	10 sec

Table 1: Topex/Poseidon orbit and receiver parameters.

Given that this strategy relies solely on measurement strength, the availability of data is first investigated. Figure 3 shows the number of satellites tracked and the Geometric Dilution of Precision (GDOP) for the data set after the data were passed through the quality control filters. The average number of satellites tracked is 4.9. Based on previous simulations [Bisnath and Langley, 1999] this value is low and is due to the limited number of receiver channels. The mean GDOP is 6.5, which indicates rather poor average geometry.

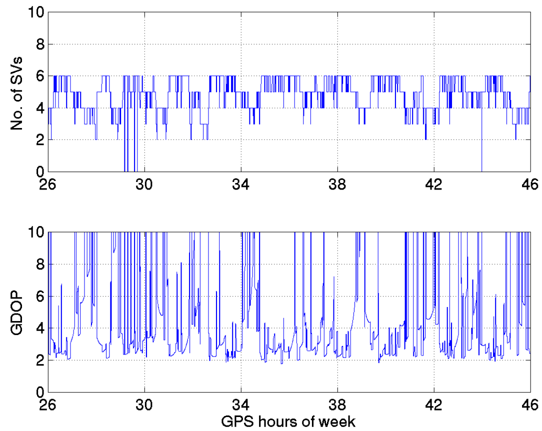

Figure 3: Number of Space Vehicles (SVs) and the Geometric Dilution of Precision (GDOP) for the Topex/Poseidon data set.

The numerous, dispersed GDOP spikes results in frequent re-initialization of the filter. This can be seen in Figure 4. In this figure our solution is differenced from the JPL solution. The latter is reportedly accurate to approximately the two-decimetre level. The 3d.r.m.s. errors for the pseudorange, filter, and smoother results are 196 cm, 117 cm, and 100 cm, respectively. Viable solutions are available for 82% of the data set. Since re-initialization means the use of the pseudorange-only solution, the estimates for this data set rely greatly on the precision of these measurements. This is not the ideal situation and solutions using terrestrial data have shown accuracies at the few decimetres 3d.r.m.s. level [Bisnath and Langley, 2001].

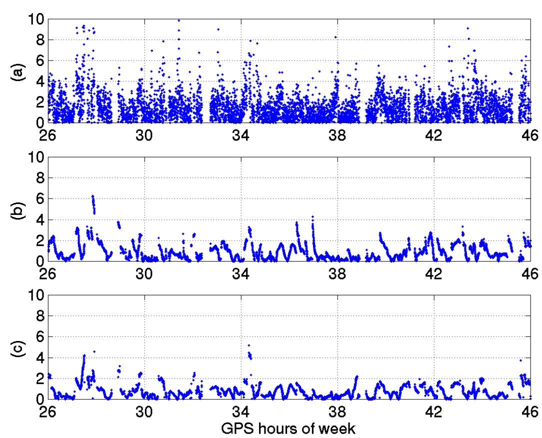

Figure 4: 3d.r.m.s. error (m) in position estimates for Topex/Poseidon data set. (a) Pseudorange solution. (b) Forward filter solution. (c) Smoothed solution.

The residuals from the forward filter estimation process are given in Figure 5. The r.m.s. of the pseudorange residuals is 78 cm and for the phase-differences is 5 cm. The phase-difference outliers are a result of filter re-initialization.

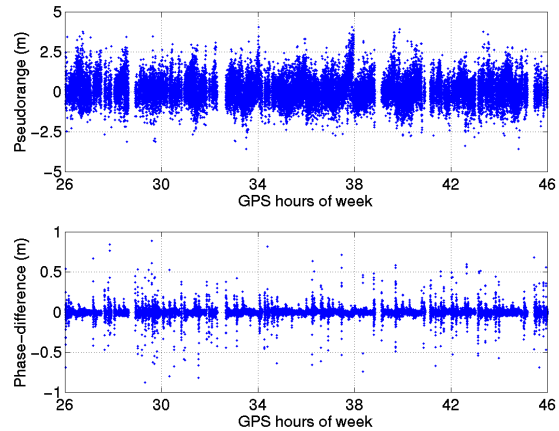

Figure 5: Forward filter observable residuals for Topex/Poseidon data set.

For an indication of how well this strategy can perform under reasonably good conditions using a portion of this data set, Figure 6 depicts the results shown in Figure 4, but isolating the time interval 36.5 hours to 37.5 hours. Even though the filter needs to re-initialize twice, the 3d.r.m.s. of the smoothed result is 33 cm with solutions available 92% of the time during the one hour interval. This data availability level is sufficient to accurately bridge the solution gaps with Lagrangian interpolation [Bisnath and Langley, 2001].

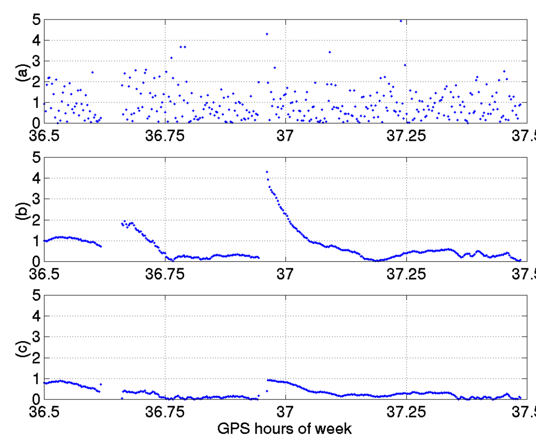

Figure 6: 3d.r.m.s. error (m) in position estimates for selected arc segment of Topex/Poseidon data set. (a) Pseudorange solution. (b) Forward filter solution. (c) Smoothed solution.

CHAMP Data

The one day of CHAMP data were collected on 7 August 2000. A description of the pertinent orbit and receiver parameters is given in Table 2. Again, the appropriate IGS precise GPS constellation orbit and clock offset file was the only additional input to the processing.

Parameter	Description
Launch date	15 July 2000
Nominal altitude	450 km
Inclination	87°
Receiver type	BlackJack
No. of SVs tracked	Up to 7
Provided observables	L1, L2,C1,P1,P2,LP1, SA,S1,S2
Provided data interval	10 sec

Table 2: CHAMP orbit and receiver parameters.

Its was expected that since the CHAMP receiver is newer and has more channels than the Topex/Poseidon receiver the geometric strength of this data set would be greater. The number of tracked satellites and the GDOP are shown in Figure 7. An average of 6.6 satellites are tracked during the 20-hour interval, with a mean GDOP of 3.1. Therefore the CHAMP data set does represent significantly better measurement strength than does the Topex/Poseidon data set; however, GDOP spikes throughout the data set again indicates that frequent filter re-initialization will be required.

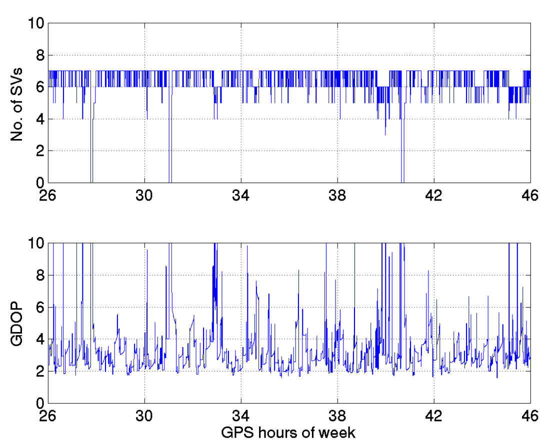

Figure 7: Number of Space Vehicles (SVs) and the Geometric Dilution of Precision (GDOP) for the CHAMP data set.

To complicate the processing, what appears to be weak data tracking occurs on data from satellites rising or setting with respect to CHAMP. This is illustrated in Figure 8. The geometry-free phase combination is primarily affected by cycle slips, the ionosphere, and dynamic tracking error. The widelane phase minus narrowlane pseudorange combination is mainly affected by cycle slips, pseudorange noise and multipath, and dynamic tracking error. The measurements therefore needed to be removed and the effect on measurement strength was considerable. The mean number of satellites tracked was reduced to 4.7 and the mean GDOP increased to 17.6. JPL processing [Kuang *et al.*, 2001] of this data set required a 25° elevation angle mask. For our processing, we used a 15° mask to retain as much data as possible, and then applied aggressive outlier detection routines.

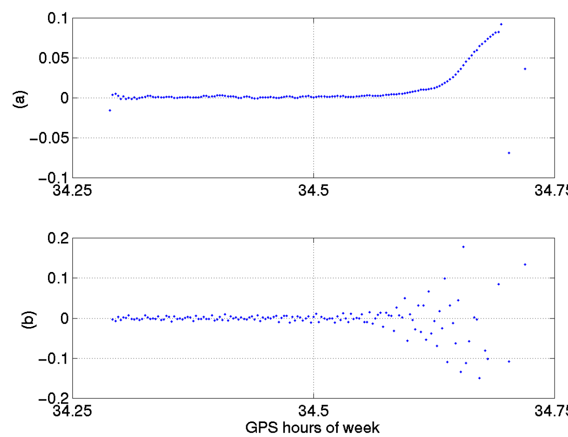

Figure 8. Carrier-phase outliers identified in linear combinations of PRN03 observables in CHAMP data set. (a) Geometry-free phase rate-of-change (m/s). (b) Widelane phase minus narrowlane pseudorange rate-of-change (m/s).

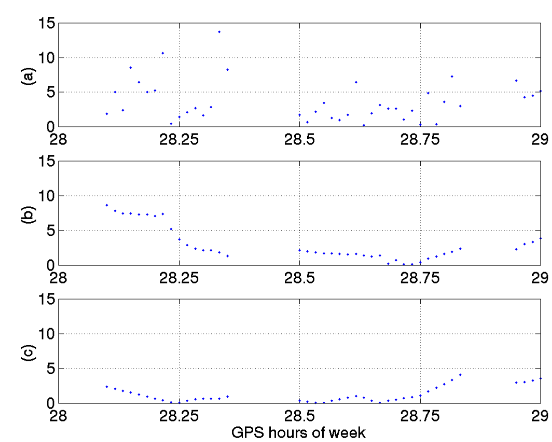

Figure 9: 3d.r.m.s. error (m) in position estimates for selected arc segment of CHAMP data set. (a) Pseudorange solution. (b) Forward filter solution. (c) Smoothed solution.

As can be imagined the geometric solution for this data set is significantly worse than that for Topex/Poseidon. Figure 9 illustrates the total displacement error as compared to initial GeoForschungsZentrum (GFZ) estimates [GFZ, 2001] for a good one hour arc. The 3d.r.m.s. errors for the pseudorange, filter, and smoother results are 6.37 m, 3.93 m, and 1.68 m, respectively. Solutions were available 69% of the time.

CONCLUSIONS

An *a posteriori* LEO orbit determination strategy based solely on GPS measurements has been devised, which is simple and efficient. The strategy incorporates a kinematic, sequential least squares filter/smoother that utilizes the full potential of the GPS measurements, and makes use of readily available GPS data products. As a by-product of the technique's design, its dynamics-free nature allows for it to be applied to GPS data from any platform.

Tests carried out with data from the Topex/Poseidon receiver indicate that approximately 100 cm 3d.r.m.s. accuracy is attainable. For periods of good geometry, this accuracy improves considerably to the 30 cm level, and point estimate gaps are small enough to be accurately interpolated. The results for the newer CHAMP receiver were much less favourable with good arcs producing approximately 170 cm 3d.r.m.s. errors. This is due to large sections of poor quality data being deleted from the data set.

FURTHER RESEARCH

A number of processing and modelling capabilities are required to refine the present strategy and allow for the most accurate position estimates. Modelling of earth rotation and phase wind-up will be included to account for these few-centimetre effects. High-rate IGS GPS clock offset files will be ingested in the processing software. Stochastic modelling will be applied to the pseudorange-only estimation. And finally, pseudorange multipath which adversely affects the filter will be mitigated through measurement de-weighting via multipath monitoring.

ACKNOWLEDGEMENTS

This research was conducted under a grant from the Natural Sciences and Engineering Research Council of Canada. The authors would like to thank Bruce Haines from the Jet Propulsion Laboratory for providing the Topex/Poseidon observables and ephemeris data.

REFERENCES

Bisnath, S.B. and R.B. Langley (1999). "Precise, Efficient GPS-based Geometric Tracking of Low Earth Orbiters." *Proceedings of the 55th Annual Meeting of the Institute of Navigation*, 28-30 June, Cambridge, Mass., U.S.A., The Institute of Navigation, Alexandria, Virginia, U.S.A., , pp. 751-760.

Bisnath, S.B. and R.B. Langley (2001). "Precise Orbit Determination of Low Earth Orbiters with GPS Point Positioning." *Proceedings of the National Technical Meeting of the Satellite Division of the Institute of Navigation*, Long Beach, California, U.S.A., 22-24 January, The Institute of Navigation, Alexandria, Virginia, U.S.A., In press.

GFZ (2001). "CHAMP Home Page." http://op.gfz-potsdam.de/champ/index_CHAMP.html. Accessed January 2001.

Hatch, R. (1982). "The synergism of GPS code and carrier measurements." *Proceedings of the Third International Geodetic Symposium on Satellite Doppler Positioning*", DMA, NOS, Las Cruces, New Mexico, 8-12 February, Physical Science Laboratory, New Mexico State University, Las Cruces, New Mexico, U.S.A., Vol. II, pp. 1213-1232.

Kleusberg, A., Y. Georgiadou, F. van den Heuvel, and P. Heroux (1993). "*GPS Data Preprocessing with DIPOP 3.0*," internal technical memorandum, Department of Surveying Engineering (now Department of Geodesy and Geomatics Engineering), University of New Brunswick, Fredericton, 84 pp.

Kleusberg, A. (1986). "Kinematic relative positioning using GPS code and carrier beat phase observations," *Marine Geodesy*, Vol. 10, No. 3/4, pp. 257-274.

Kuang, D., Y. Bar-Sever, W. Bertiger, S. Desai, B. Haines, B. Iijima, G. Kruizinga, T. Meehan, and L. Romans (2001). "Precise Orbit Determination for CHAMP using GPS Data from BlackJack Receiver." *Proceedings of the National Technical Meeting of the Satellite Division of the Institute of Navigation*, Long Beach, California, U.S.A., 22-24 January, The Institute of Navigation, Alexandria, Virginia, U.S.A., In press.

Neilan, R.E., J.F. Zumberge, G. Beutler, and J. Kouba (1997). "The International GPS Service: A global resource for GPS applications and research." *Proceedings of the 10th International Technical Meeting of the Satellite Division of the Institute of Navigation*, Kansas City, Missouri, U.S.A., 16-19 September, The Institute of Navigation, Alexandria, Virginia, U.S.A., pp. 883-889.

Yunck, T. P., S.-C. Wu, and J.-T. Wu (1986). "Strategies for sub-decimeter satellite tracking with GPS," *Proceedings of IEEE Position, Location, and Navigation Symposium 1986*, Las Vegas, Nevada, U.S.A., 4-7 November, The Institute of Electrical and Electronics Engineers, Inc., New York, New York, U.S.A., pp. 122-128.