KALMAN-FILTER-BASED GPS AMBIGUITY RESOLUTION FOR REAL-TIME LONG-BASELINE KINEMATIC APPLICATIONS

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Abstract

Resolving the GPS carrier-phase ambiguities has been a continuing challenge for sub-centimeter-level high-precision GPS positioning. In kinematic and long-baseline applications, the challenge turns out to be even greater due to the substantial problems in the observation time series – the decorrelation of biases, the quasi-random behavior of multipath and the correct interpretation of receiver system noise for observations conducted in kinematic mode – which can be ignored to some degree in static and short-baseline applications. As baseline lengths grow longer, eventually, these problems will make it difficult to get reliable ambiguity solutions in kinematic applications.

We have found that the problems related to kinematic long-baseline applications can be handled in an optimal way when a particular generalized procedure is adopted in the observations processing scheme. The generalized procedure includes: a functional model which takes into account all significant biases; a stochastic model which is derived directly from the observation time series; a quality control scheme which handles cycle slips (or outliers); and a parameter-estimation scheme which includes a simultaneous ambiguity search process. The prototype approach described in this paper follows the generalized procedure. For each stage of the procedure, the new concepts of our approach are explained and some preliminary test results are given.

1. Introduction

It has been a continuing challenge to determine and fix the GPS carrier-phase ambiguities, especially for long-baselines. Moreover, the challenge is even greater for kinematic GPS applications. Generally, the difficulty in solving the ambiguities is due to the decorrelation of biases in the GPS observations. As is well known, the GPS observations at the base and remote stations will be influenced by different atmospheric effects and satellite orbit bias as the baseline length between the stations gets longer. Furthermore, when the pseudorange observations are incorporated with the carrier-phase observations, multipath can be the dominant error source that makes it difficult to solve the ambiguities because of its quasi-random behaviour over a relatively short time span. In kinematic situations, it is not easy to model the observation noise since the dynamics of a moving platform may mask some aspects of the observation noise which usually can be well modeled statistically by an elevation-angle dependent function.

To obtain optimal solutions in the least-squares estimation, a functional (or deterministic) and a stochastic model should be specified correctly. A functional model describes the relationship between observations and unknown parameters while a stochastic model represents the noise characteristics of the observations. Actually, the challenge that we face in long-baseline kinematic applications is how to correctly specify the models without ignoring the problems mentioned above; i.e., the decorrelation of biases, the quasi-random behavior of multipath and the receiver system noise (or observation noise) for observations conducted in kinematic mode. In this case, the problem related to the functional model is that the number of unknown parameters is greater than that of the observations, when constraining external observations such as those provided by an external atomic clock, inertial navigation system (INS) and so on are not available. Furthermore, it turns out to be very difficult to specify a correct stochastic model if we opt for a simpler functional model by ignoring certain parameters because of the residual effects of these parameters as well as the dynamics of a moving platform. As a fundamental problem in processing the GPS observations, we also face a quality control issue; i.e., how do we implement a robust cycle-slip (or outlier) handling routine? Especially for long-baseline kinematic applications, this issue turns out to be another challenge.

Basically, our approach in attempting to meet these challenges follows a generalized procedure which consistently keeps track of the noted problems in long-baseline kinematic applications. The prototype approach described in this paper is based on the case when dual-frequency GPS observations are available. In situations where external observations are also available, the approach can integrate the additional information without undue complexity.

2. Considerations for a Reliable Approach

As has been experienced, the stochastic model is typically more difficult to handle than the functional model when considering a reliable approach for long-baseline kinematic applications. Assuming that the functional model includes all significant unknown parameters (e.g., those associated with atmospheric effects, satellite orbit bias, multipath and so on) except for receiver system noise (i.e., antenna noise, cable loss and receiver noise; see Langley (1997)), we can deal with the stochastic model more easily. In this case, the problems associated with the stochastic model are: cross correlation (between different observation types), time correlation (between epochs), spatial correlation (between channels), elevation-angle dependence and the error probability distribution (see Tiberius et al. (1999)). Basically, we assume that all parameters describing the stochastic modeling problems can be calibrated in the laboratory. As long as the functional model is correct, these parameters can be used at a remote site without tuning. However, it should be noted that the elevation-angle dependence of the system noise often varies with the particular kinematic situation. The elevation-angle dependence of the system noise is induced mainly by the receiver antenna's gain pattern, with other factors such as atmospheric signal attenuation. The elevation angle is normally computed with respect to the local geodetic horizon plane at the antenna phase center regardless of the actual orientation of the antenna. Accordingly, the relationship between antenna gain and the signal elevation angle may be difficult to assert when the antenna orientation is changing which can happen often in kinematic situations (see Langley (1997) for more extensive discussion).

If we use a functional model which includes all significant unknown parameters, we will face a problem in conventional least-squares estimation or Kalman filtering; i.e., the singularity or observability problem. Although this problem can be handled at second-hand by a parameter transformation method to reduce the number of the unknown parameters (Jin, 1996), an inherent difficulty still remains: the parameter estimates can be biased because the higher-frequency components of the unknown parameters (e.g., ionospheric scintillation, jerk and so on) cannot be estimated under the rank-deficiency condition. In using a Kalman filter, we can augment higher-order time derivatives of the state vector with the unknown parameters in order to take into account the higher-frequency components. However, the state estimates still can be biased. In the context of signal processing, this problem is related to the sampling rate (Ifeachor and Jervis, 1993). For example, if we have an observation time series recorded at a one second sampling interval (i.e., 1 Hz sampling rate), the time series can contain information in a bandwidth of 0.5 Hz (i.e., the Nyquist frequency). To obtain a wide bandwidth which includes higher-frequency components, we have to increase the sampling rate. By how much should we increase it? The quick answer is enough to remove the effect of aliasing in order to get unbiased parameter estimates.

3. Kalman-Filter-Based Ambiguity Search Process

There can be many approaches to the question: Which strategy will be preferable for handling GPS observations in long-baseline kinematic applications? However, in terms of implementation, our answer is a Kalman filter approach combined with an ambiguity search method which can deal with both the functional and stochastic models in an optimal way (Fig. 1).

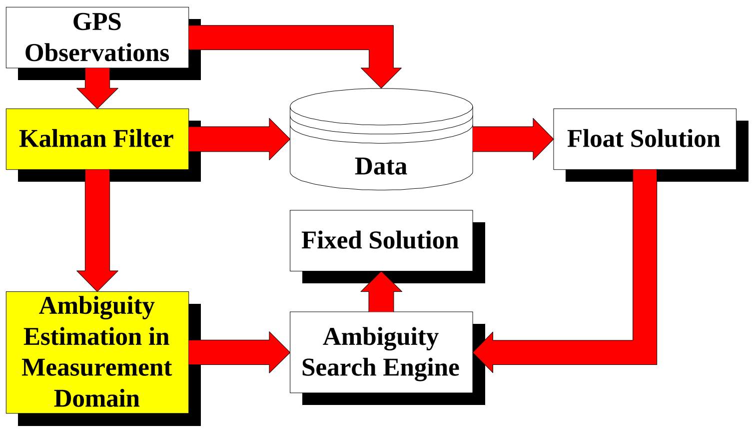

Fig. 1. Functional block diagram to fix the GPS ambiguities in long-baseline kinematic applications.

3.1 Kalman Filter Approach

We have found that a Kalman filter approach can efficiently implement quality control schemes such as cycle-slip handling (i.e., cycle-slip detection, identification and adaptation), and that the parameter estimates of the filter can be used at second-hand in the ambiguity search process as long as the parameter estimates are not biased. However, fundamental concerns related to its implementation are: 1) How do we reduce the number of unknown parameters in the filter state vector? 2) How do we ensure the observability of the given system model under the rank-deficiency condition? 3) Which implementation method is most efficient?

Basically, the problem is that the number of unknown parameters is much greater than that of the observations. This is an inherent problem of carrier-phase applications and turns out to be a substantial one in such an approach as ours which tries to estimate all the bias parameters (so far, all except for the multipath in the carrier-phase observations). To reduce the number of unknown parameters, the double differencing scheme is used in our approach. In addition, dual-frequency carrier phases (L1 and L2) and code pseudoranges (P1 and P2, or C/A and P2) are used to increase observation redundancy. Furthermore, the unknown parameters are transformed to ensure the observability of the given system model. A separate Kalman filter is implemented for each double-difference time series because its programming and stochastic modeling are easier. As a result, we form the following state vector:

$$\hat{\mathbf{x}} = \left[\hat{L} \quad \cdots \quad \hat{I} \quad \cdots \quad \hat{B}_1 \quad \cdots \quad \hat{B}_2 \quad \cdots \quad \hat{n}_1 \quad \hat{n}_2 \right]^T \qquad (1)$$

with

$$\hat{L} = L + \frac{1}{g-1}(g\,b_1 - b_2) + \frac{1}{g-1}\left(g\,B_1'^0 - B_2'^0\right)$$
$$\hat{I} = I + \frac{1}{g-1}(b_1 - b_2) - \frac{1}{g-1}\left(B_1'^0 - B_2'^0\right)$$
$$\hat{B}_1 = B_1' - B_1'^0$$
$$\hat{B}_2 = B_2' - B_2'^0$$
$$\hat{n}_1 = n_1 + \frac{1}{g-1}\left[-(g+1)B_1'^0 + 2B_2'^0\right]$$
$$\hat{n}_2 = n_2 + \frac{1}{g-1}\left[-2g\,B_1'^0 + (g+1)B_2'^0\right]$$

and

$$B_1' = B_1 + \frac{1}{g-1}\left[-(g+1)b_1 + 2b_2\right]$$
$$B_2' = B_2 + \frac{1}{g-1}\left[-2g\,b_1 + (g+1)b_2\right]$$
$$B_1'^0 = B_1^0 + \frac{1}{g-1}\left[-(g+1)b_1^0 + 2b_2^0\right]$$
$$B_2'^0 = B_2^0 + \frac{1}{g-1}\left[-2g\,b_1^0 + (g+1)b_2^0\right], \quad (2)$$

where L stands for the combined effect of a priori (or assumed) geometric range, satellite orbit bias and tropospheric delay $(L = r + s + t)$; I for the ionospheric delay; b for the multipath in carrier phases; B for the multipath in code pseudoranges; n for the ambiguities (in distance units); "$\cdots$" for the higher-order time derivatives of the parameters; a constant $g = (l_2 / l_1)^2$; superscript "0" for the initial (at the start of observations) bias value; subscripts "1" and "2" correspond to L1 and L2, P1 and P2 (or C/A and P2), respectively. The "^" symbol indicates a transformed parameter.

It should be noted that each transformed parameter of Eq. (2) includes a true parameter value, the carrier-phase multipath on L1 and L2, constant initial-multipath ($B_1'^0$ and $B_2'^0$), and receiver system noise. The constant initial-multipath can be separated from the parameter estimates in the ambiguity search process (see section 3.4) but the carrier-phase multipath is so far difficult to estimate in this approach as long as we cannot use additional observations such as the signal-to-noise ratio (SNR) for the carrier-phase observations. It should be pointed out that the parameter estimates determined in this way could be biased in some cases as mentioned previously in section 2. In addition, we will not get unbiased parameter estimates if cycle slips are not handled perfectly.

3.2 Quality Control

Since we do not consider a cycle slip as an unknown parameter in the functional model, we have to detect and remove it from the observations. If we fail to do that, the Kalman filter parameter estimates will be biased after all. Basically, we have used a cycle-slip handling procedure similar to that of Teunissen (1990); i.e., the DIA (Detection, Identification and Adaptation) procedure based on the Kalman filter prediction residuals. However, we have found that the procedure does not work as well as expected in kinematic situations. This problem is due to the dynamics of a moving platform and eventually, related to sampling rate.

To fortify the procedure against platform dynamics, we use a masking technique based on a logical intersection of necessary and sufficient conditions for cycle-slip detection and identification. When a cycle-slip happens, we can see a certain spike in the quadruple-difference (obtained by differencing consecutive triple-difference observations) time series. This provides a necessary condition for cycle-slip identification. As a conventional approach incorporated within a Kalman filter, we can use prediction residuals to detect a cycle slip. However, this should be used carefully because the prediction residuals are very sensitive to the dynamics of a moving platform and the sampling rate of the observations. Another approach given in Kee et al. (1997) is the use of the ionospheric-delay drift estimates. However, this also should be used carefully because there are cases when a cycle slip cannot be detected such as when cycle slips of the same size (in distance units) occur simultaneously on L1 and L2, not to mention the very obvious case when cycle slips in both carrier phases cancel each other in the ionosphere-free combination (i.e., $\frac{1}{77}c1-\frac{1}{60}c2=0$, where *c1* and *c2* represent cycle slips on L1 and L2 in cycle units). Nevertheless, in a wide sense, these two approaches – prediction residuals and ionospheric-delay drift estimates – provide sufficient conditions for detecting cycle slips.

So far, we have found that the performance of this procedure is almost perfect as far as cycle-slip detection and identification are concerned. However, cycle-slip adaptation (i.e., removing a cycle slip from the observations) should be executed carefully because the size of the cycle slip must be determined correctly to remove it. If we try to determine the size of a cycle slip using the Kalman filter prediction residuals, we may introduce a new bias in the observations. As a simple strategy to avoid this problem, we can reset the Kalman filter state vector whenever a cycle slip is detected and identified.

3.3 Receiver System Noise Estimation

If the stochastic model is not correct, it will affect the computed parameter estimates to some degree. As was mentioned already, we had better not use the elevation-angle dependent function for the stochastic model for data collected from moving platforms. An alternative, and potentially more powerful, approach can be derived directly from measurements of the quality of each carrier-phase observation. This information is contained in the SNR (or alternatively in the carrier-to-noise-power-density ratio, C/N_0). Although the potential merits of using the SNR information as a stochastic modeling scheme was already discussed in Talbot (1988), a comprehensive examination of the technique has only taken place recently (Hartinger and Brunner, 1998; Barnes et al., 1998). Although some GPS receiver manufacturers provide SNR values in their data streams, meaningful SNR values are not easy to come by (Collins and Langley, 1999).

The following concept represents another alternative approach which can be derived directly from the observation time series. We use the quintuple-difference (differencing consecutive quadruple-difference observations after deleting cycle-slip spikes) time series to estimate the receiver system noise for observations conducted in kinematic mode. We have chosen this approach because the quadruple-difference time series are already obtained for quality control as described in section 3.2. Therefore, this approach can be implemented without undue complexity. In this approach, we assume that the effects of the unknown parameters (except the receiver system noise) are removed in the quintuple-difference time series. For example, consider the L1 quintuple-difference carrier-phase time series

$$\dddot{\Phi}_1 = \dddot{r} + \dddot{t} + \dddot{s} - \dddot{I} + \dddot{b}_1 + \dddot{n}_1 + \dddot{e}_1 \,, \tag{3}$$

where Φ_1 is the L1 double-difference observable. Using the one-dimensional Taylor series including higher-order time derivatives for each unknown parameter, we have

$$S(t) = S(t_0) + S'(t_0)(t-t_0) + \frac{1}{2}S''(t_0)(t-t_0)^2 + \frac{1}{6}S'''(t_0)(t-t_0)^3 + R(t) \,, \tag{4}$$

where *S* represents each unknown parameter and *R* is a remainder term known as the Lagrange remainder. Assuming that the observation time interval $(t-t_0)$ is the same as Ä for the time series, we have the following quintuple-difference:

$$\begin{aligned} \dddot{S}(t_3) &= S(t_3) - 3\cdot S(t_2) + 3\cdot S(t_1) - S(t_0) \\ &= S'''(t_0)\Delta^3 + \textstyle\sum_R (t_3) \,, \end{aligned} \tag{5}$$

where $\sum_R (t_3)$ is the quintuple-difference for the remainder *R*. Substituting Eq. (5) into (3) gives

$$\dddot{\Phi}_1(t_3) = \left[\sum_{\forall S} S'''(t_0)\right]\Delta^3 + \sum_{\forall S}[\Sigma_R(t_3)] + \dddot{e}_1(t_3)\ , \tag{6}$$

where

$$\sum_{\forall S} S'''(t_0) = [\,r''' + t''' + s''' - I''' + b_1''' + n_1'''](t_0)\ . \tag{7}$$

If the effect of the terms in the right-hand side of Eq. (7) is small enough to be ignorable and/or the sampling rate (1/Ä) is high in Eq. (6), and if the effect of the second term in the right-hand side of Eq. (6) (i.e., the effect of the quintuple-difference for the remainder *R*) is also small enough to be ignorable, we can get an acceptable inference as:

$$\dddot{\Phi}_1 \approx \dddot{e}_1\ . \tag{8}$$

3.4 Ambiguity Search Process

Using the estimates of the state vector, we can transform the original carrier-phase double-difference observations to those to be used for the ambiguity search process. The purpose of this transformation is to reduce the number of unknown parameters at the ambiguity search step. However, there can be some cost to pay for this transformation (i.e., the receiver system noise is increased and time-correlated). We use an ionosphere-free transformation to reduce this cost:

$$\begin{aligned} \Phi_1 + \ \hat{I} &= L + \frac{1}{g-1}(g\,b_1 - b_2) + n_1 - \frac{1}{g-1}\left(B_1'^0 - B_2'^0\right) + e_1' \\ \Phi_2 + g\,\hat{I} &= L + \frac{1}{g-1}(g\,b_1 - b_2) + n_2 - \frac{g}{g-1}\left(B_1'^0 - B_2'^0\right) + e_2'. \end{aligned} \tag{9}$$

As a matter of fact, we have found that the transformed observations are similar to the ionosphere-free linear combination but have smaller observation noise. The time-correlated observation noise can be estimated using the variance-covariance matrix which is obtained adaptively from the Kalman filter. Using the transformed observation for each double-difference observation, we have the augmented observation equations as

$$\begin{aligned} \mathbf{\ddot{O}_1} + \ \mathbf{\hat{I}} - (\mathbf{\tilde{n}_0} + \mathbf{\hat{o}_0}) &= \mathbf{Ax} + \mathbf{\ddot{E}_1 N_1} + \mathbf{s} + \frac{1}{g-1}(g\,\mathbf{b_1} - \mathbf{b_2}) - \frac{1}{g\text{ -}1}\mathbf{B}_{\mathbf{12}}'^{\mathbf{0}} + \mathbf{\mathring{a}}_1' \\ \mathbf{\ddot{O}_2} + g\,\mathbf{\hat{I}} - (\mathbf{\tilde{n}_0} + \mathbf{\hat{o}_0}) &= \mathbf{Ax} + \mathbf{\ddot{E}}_2\mathbf{N_2} + \mathbf{s} + \frac{1}{g-1}(g\,\mathbf{b_1} - \mathbf{b_2}) - \frac{g}{g\text{ -}1}\mathbf{B}_{\mathbf{12}}'^{\mathbf{0}} + \mathbf{\mathring{a}}_2'\ , \end{aligned} \tag{10}$$

where **x** includes unknown baseline components and the residual tropospheric delay; **A** is the corresponding design matrix for **x**; **N** is the ambiguity vector (in cycle units); **Ë** is the corresponding design matrix for **N**; **s** is the satellite orbit bias vector; **b** is the carrier phase multipath vector; $\mathbf{B}_{12}'^{0} = \mathbf{B}_1'^{0} - \mathbf{B}_2'^{0}$ is a constant vector; $\mathbf{\tilde{n}_0}$ and $\mathbf{\hat{o}_0}$ are the initial estimates of the geometric range and the tropospheric delay, respectively. In practical implementation of Eq. (10), we assume that the carrier-phase multipath is ignorable and precise satellite orbit is available. As can be understood in looking at Eq. (10), the ambiguities cannot be separated from the parameters $\mathbf{B}_{12}'^{0}$ because they are also constant. This problem can be solved using the widelane combination of the estimates $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ in distance units. From Eq. (2), we can formulate the widelane combination as

$$\hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_2 = \left(\Lambda_1 \mathbf{N}_1 - \Lambda_2 \mathbf{N}_2\right) - \mathbf{B}_{12}^{\prime 0}. \tag{11}$$

As the Kalman filter converges, the widelane combination verges to a constant quickly. This means that the parameters $\mathbf{B}_{12}^{\prime 0}$ can be determined in the ambiguity search process, where $\mathbf{N}_1$ and $\mathbf{N}_2$ are given as known values, as long as the widelane combination, $\hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_2$, converges.

For the ambiguity search process, we use the independent-ambiguity-search approach (Hatch, 1990). Since there remain four unknown parameters in the observation equations after the observations are transformed, we always have eight search levels (four search levels for $\mathbf{N}_1$ and $\mathbf{N}_2$, the L1 and L2 ambiguities, respectively) regardless of the number of double-difference observations. In this case, the search space may be enormous even if a small search window is used. This means that the ambiguity search process may be so time-consuming that it is not appropriate for a real-time system. In order to overcome this problem, we use an efficient ambiguity search engine, namely OMEGA (Optimal Method for Estimating GPS Ambiguities) as described by Kim and Langley (1999a and 1999b).

4. Preliminary Test Results

We have tested our technique using a marine kinematic data set. The dual-frequency data were recorded at a one second sampling interval on board a hydrographic sounding ship at Trois-Rivières, on the St. Lawrence River, 130km upstream (southwest) of Québec City, on 22 October 1998 and simultaneously at one reference station (Trois-Rivières DGPS) in the Canadian Coast Guard (CCG) DGPS and OTF network (Fig. 2).

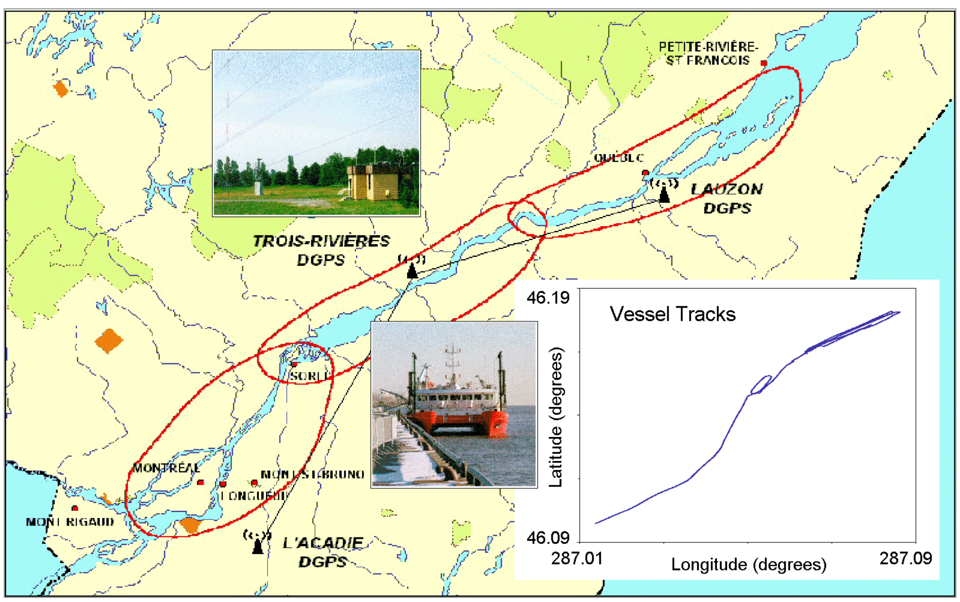

Fig. 2. Canadian Coast Guard DGPS and OTF network: coverage of the St. Lawrence River. Test data were recorded at the Trois-Rivières reference station and on a hydrographic sounding ship on October 22, 1998.

Figure 3 shows the performance of the Kalman filter. Note that each parameter estimate includes the carrier-phase multipath on L1 and L2, the constant initial-multipath, and the receiver system noise. The most significant difference between the parameter estimates and true parameter values is an offset along the y-axis due to the constant initial-multipath.

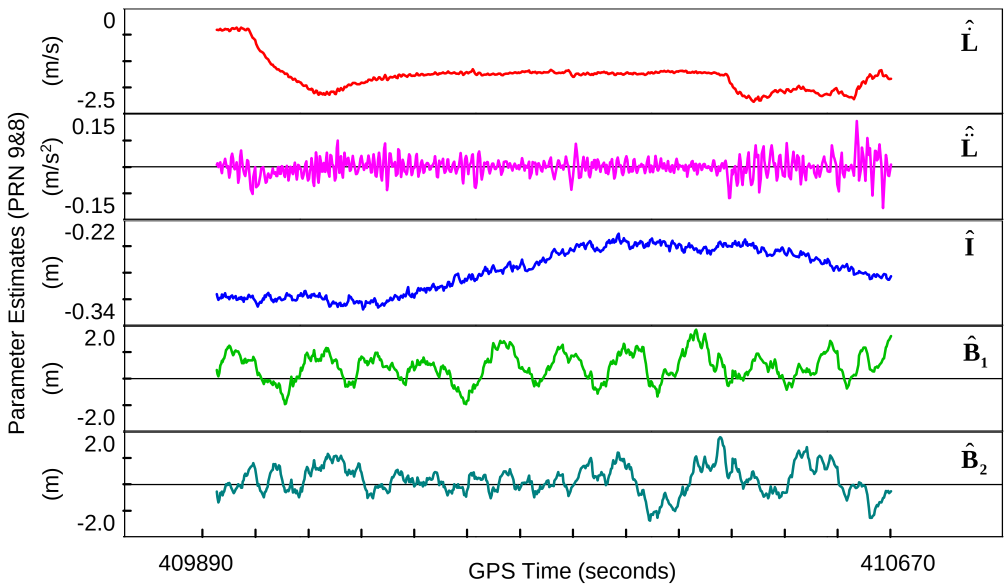

Fig. 3. Kalman filter parameter estimates for the double-difference time series of PRN8 and PRN9.

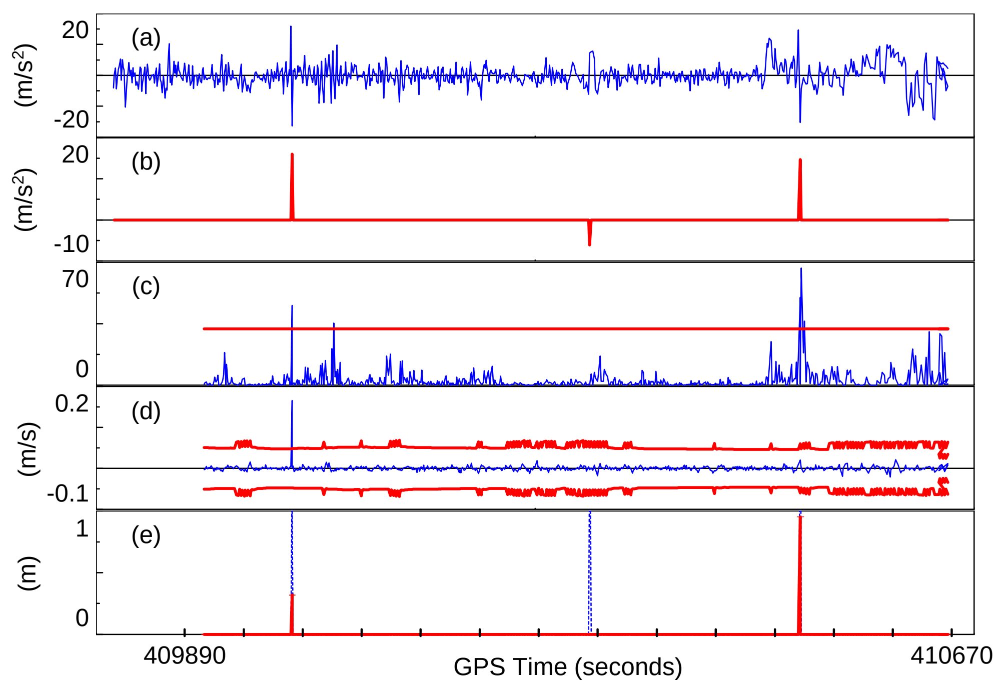

Fig. 4. Example of cycle-slip detection and identification procedures (PRN15&30): (a) L1 Quadruple-difference time series; (b) Cycle-slip candidates detected by spikes; (c) Cycle-slip candidates detected by the Kalman filter prediction residuals (95% confidence level); (d) Cycle-slip candidates detected by the ionospheric-delay drift estimates (95% confidence level); and (e) Masking results (cycle-slip identification).

Figure 4 shows the cycle-slip detection and identification procedures. The performance of these procedures (Fig. 4e) is greatly improved compared with the conventional approaches as shown in Fig. 4c and 4d. For example, Fig. 4e shows perfect cycle-slip detection and identification results (i.e., there were two cycle slips on the observation time series used). On the other hand, Fig. 4c shows that the approach using the Kalman filter prediction residuals falsely detected cycle slips at certain epochs. If we set the confidence level lower than 95%, the results will be even worse. Furthermore, Fig. 4d shows that the approach using the ionospheric-delay drift estimates did not detect a cycle slip at a certain epoch where, in fact, a cycle slip did occur.

In Fig. 5 and 6, we can see how the dynamics of a moving platform can mask the behaviour of the receiver system noise. The double-difference receiver system noise was estimated using the observation time series for 15 minutes. For the present, the performance comparison between the quintuple-difference approach and the SNR approach has not been conducted. A more detailed analysis will be given in near future.

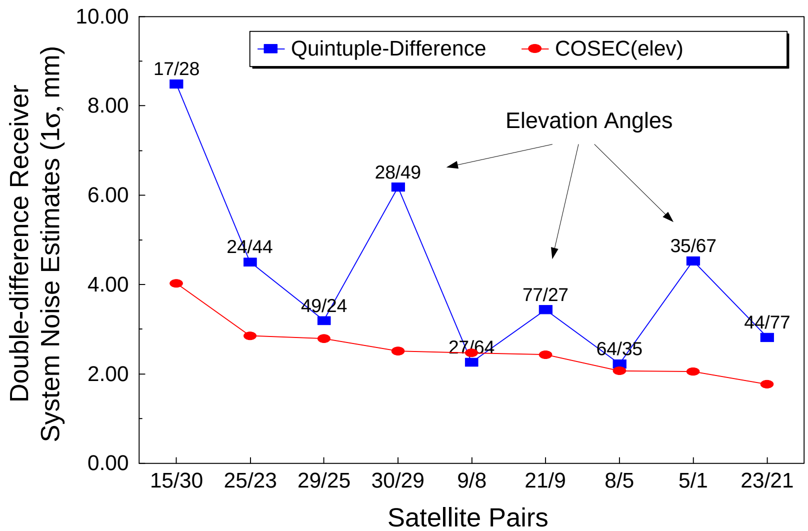

Fig. 5. Receiver system noise estimates comparison between the quintuple-difference approach and an elevation-angle dependent function (1/sin(elev)).

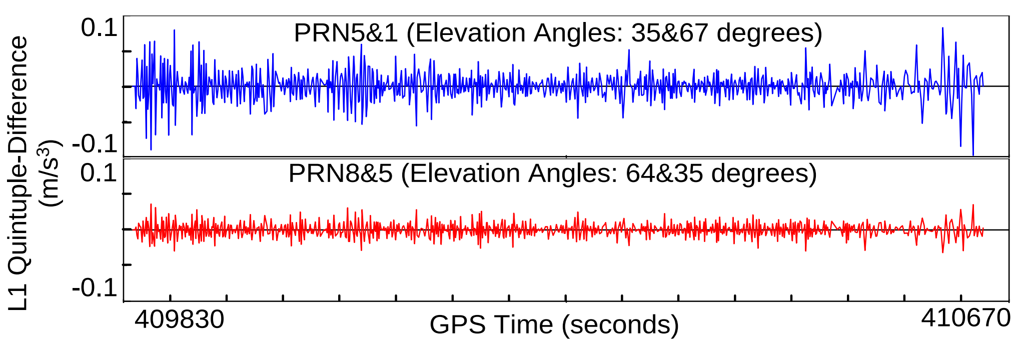

Fig. 6. Quintuple-difference time series for estimating the receiver system noise.

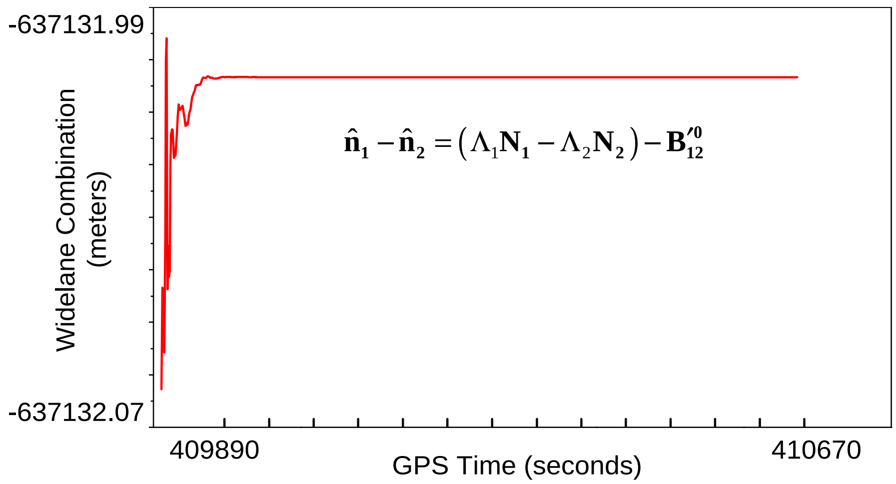

Fig. 7. Widelane combination of the Kalman filter estimates for estimating the constant initial-multipath.

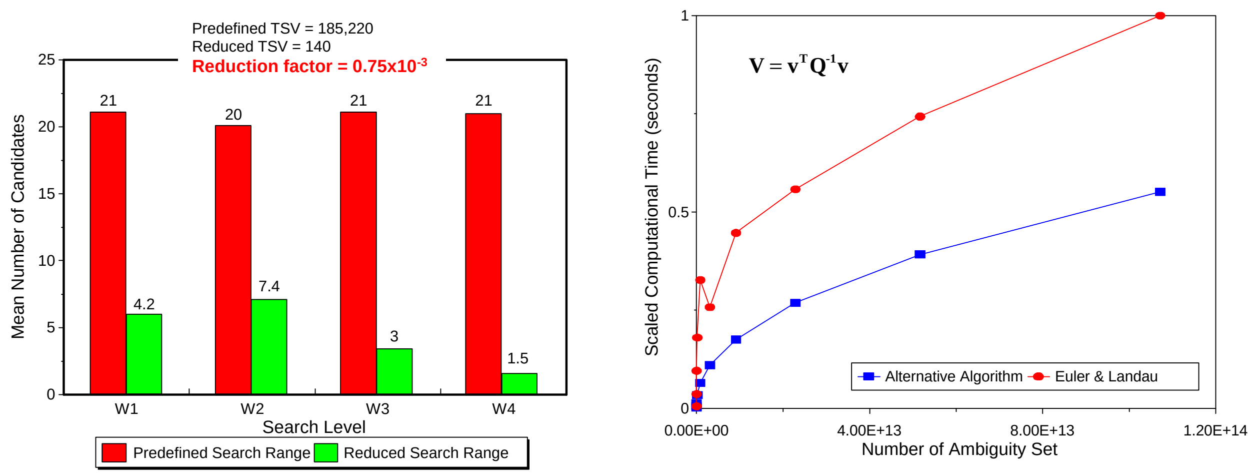

Fig. 8. Computational efficiency of the ambiguity search engine: (a) Search space reduction factor analysis using the OMEGA method and (b) Computation time comparison between an alternative algorithm and the modified Cholesky decomposition method for the computation of the quadratic form of the residuals.

Fig. 7 shows that the widelane combination in Eq. (11) becomes so constant quickly that the constant initial-multipath, $\mathbf{B}'^0_{12}$, can be determined in the ambiguity search process, where $\mathbf{N}_1$ and $\mathbf{N}_2$ are given as known values. In Fig. 8, we can see that the OMEGA method for the ambiguity search process attains great computational efficiency through total search volume (TSV) reduction (by 3 orders of magnitude) and an alternative algorithm (Kim and Langley, 1999b) for the computation of the quadratic form of the residuals (about 50% improvement compared with Euler and Landau (1992)).

5. Discussion and Conclusion

We have developed a prototype approach to solve the ambiguity fixing problems in long-baseline kinematic applications. The main feature of the technique, which may differ from other approaches, is that the system takes into account the problems of handling the decorrelation of biases, the quasi-random behavior of multipath and the receiver system noise in kinematic mode all at the same time within the functional and stochastic models for the GPS observations. In other words, we do not simply ignore these problems and hope their effects are averaged out. Instead, all the bias parameters and the receiver system noise (except multipath in the carrier-phase observations) are estimated while a software process for quality control of the observations is proceeding. Our new approach also features improved computational efficiency of the ambiguity search process by reducing the search space and the use of a new algorithm for the quadratic form of the residuals.

Acknowledgements

This work has been conducted under the GEOIDE Network of Centres of Excellence (project ENV#14). The support of the Canadian Coast Guard; the Canadian Hydrographic Service; VIASAT Géo-Technologie Inc.; Geomatics Canada; and the Centre de Recherche en Géomatique, Université Laval is gratefully acknowledged.

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