The Multipath Divergence Problem in GPS Carrier-Smoothed Code Pseudorange

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Abstract

The Hatch filter is an averaging filter that can smooth pseudorange measurements with continuous carrier phase. As is well known, if the averaging constant is increased beyond a certain point, the filter diverges because of opposite signs of the ionospheric delay in the code pseudorange and carrier phase. Moreover, the Hatch filter does not generate a confidence value for the filtered output. To overcome these problems, the divergence-free Hatch filter and the weighted Hatch filter were introduced.

One critical problem of the smoothing process involves multipath effects. If multipath occurs when the weight of the observed pseudorange is still large, then the effect of multipath will contaminate the smoothed pseudorange, not just at that epoch but also at several subsequent epochs. Therefore, the smoothing process usually has been used only at the reference stations, where multipath should be virtually non-existent in order to provide high accuracy differential corrections. For sites where multipath does exist, we often optimistically assume that after a certain time period the effect of multipath will be reduced since the variance of the smoothed pseudorange gets smaller with time. However, over short time intervals, multipath will cause the smoothing filter output to diverge.

We introduce the multipath divergence problem in this paper. After giving an equation for the multipath divergence, we propose a new procedure to overcome the multipath problem in the previous approaches.

Multipath Divergence

In some applications using the carrier-smoothed code pseudorange, if multipath in the pseudorange is not eliminated, the resulting smoothed pseudorange can diverge in a relatively short time span because of the quasi-random behaviour of multipath. When a multipath bias parameter is included in the observation equation, its contribution to the smoothed pseudorange is given as

$$\Delta B_k = \sum_{i=1}^{k} \left\{ w_{2i} b_i \prod_{j=i+1}^{k(k>1)} w_{1j} \right\} \qquad (1)$$

with the following condition:

$$if \quad (j > k) \quad then \quad \prod_{j=i+1}^{k(k>1)} w_{1j} = 1, \qquad (2)$$

where w_1 and w_2 represent the respective weighting factors for the carrier-phase and code pseudorange update in the smoothing filter; b is the multipath bias value and k is the current observation epoch. We can find two conditions for which the contribution of multipath can be ignored, i.e., when $b_i = 0$ or $w_{2i} = 0$. The first condition is usually satisfied at the reference station where multipath is eliminated a priori [*Euler and Goad*, 1991; *Kee et al*., 1997; *Hwang et al*., 1999]. As the smoothing process proceeds, we know that the second condition also can be fulfilled. However, this does not mean that, if multipath remains in the smoothing process, its contribution vanishes. Therefore, for some situations with large multipath, the resulting smoothed pseudorange may diverge if we do not reduce the impact of multipath in some way. This paper presents a summary of our approach. A more complete description of our work will be submitted for journal publication.

Double-difference Multipath Estimation

Basically, we focus on the smoothing process at a remote station operating in kinematic mode. Therefore, we cannot use approaches designed for reference stations where the multipath can be calibrated or otherwise mitigated [*Kee and Parkinson*, 1994; *Comp and Axelrad*, 1996; *Ray et al*., 1999; *Reichert and*

Axelrad, 1999]. Instead, we use a multipath estimation approach incorporating within a Kalman filter. Using dual-frequency carrier-phases (L1 and L2) and code pseudoranges (P1 and P2, or C/A and P2), we form the following state vector:

$$\hat{\mathbf{x}}_{\mathbf{k}} = \left[\hat{\mathbf{L}} \quad \cdots \quad \hat{\mathbf{I}} \quad \cdots \quad \hat{\mathbf{b}}_1 \quad \cdots \quad \hat{\mathbf{b}}_2 \quad \cdots \quad \hat{\mathbf{n}}_1 \quad \hat{\mathbf{n}}_2\right]^{\mathbf{T}}_{\mathbf{k}}, \quad (3)$$

where **L** stands for the combined geometric range and tropospheric delay $(\mathbf{L} = r + t)$; **I** for the ionospheric delay; **b** for the multipath in code pseudoranges; **n** for the ambiguities; "$\cdots$" for higher order time derivatives of the parameters; subscripts "1" and "2" correspond to L1 and L2 for $\mathbf{n}_1$ and $\mathbf{n}_2$, P1 and P2 (or C/A and P2) for $\mathbf{b}_1$ and $\mathbf{b}_2$, respectively. In our case, a separate Kalman filter is implemented for each double-difference time series. Therefore, each parameter in the state vector becomes

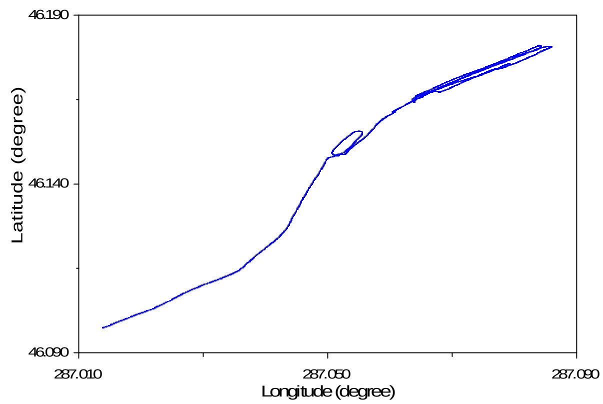
$$\begin{aligned}
\hat{\mathbf{L}}_{\mathbf{k}} &= \mathbf{L}_{\mathbf{k}} + \frac{g}{g-1}\mathbf{B}_1^{\mathbf{0}} - \frac{1}{g-1}\mathbf{B}_2^{\mathbf{0}} \\
\hat{\mathbf{I}}_{\mathbf{k}} &= \mathbf{I}_{\mathbf{k}} - \frac{1}{g-1}(\mathbf{B}_1^{\mathbf{0}} - \mathbf{B}_2^{\mathbf{0}}) \\
\hat{\mathbf{b}}_{\mathbf{1k}} &= \mathbf{B}_{\mathbf{1k}} - \mathbf{B}_1^{\mathbf{0}} \\
\hat{\mathbf{b}}_{\mathbf{2k}} &= \mathbf{B}_{\mathbf{2k}} - \mathbf{B}_2^{\mathbf{0}} \\
\hat{\mathbf{n}}_{\mathbf{1k}} &= l_1\mathbf{N}_1 - \frac{g+1}{g-1}\mathbf{B}_1^{\mathbf{0}} + \frac{2}{g-1}\mathbf{B}_2^{\mathbf{0}} \\
\hat{\mathbf{n}}_{\mathbf{2k}} &= l_2\mathbf{N}_2 - \frac{2g}{g-1}\mathbf{B}_1^{\mathbf{0}} + \frac{g+1}{g-1}\mathbf{B}_2^{\mathbf{0}},
\end{aligned} \quad (4)$$

where $\mathbf{B}^{\mathbf{0}}$ represents the initial (at the start of observations) multipath bias. In this approach, we will have the double-difference multipath estimates with the constant biases, $\mathbf{B}_1^{\mathbf{0}}$ and $\mathbf{B}_2^{\mathbf{0}}$, at each epoch. Although the multipath estimates have the initial constant biases, these do not raise a divergence problem in the smoothing filter. In fact, we can also estimate the initial constant biases with incorporation in an ambiguity resolution technique.

Test Results

We have tested our technique using a kinematic data set. The data were recorded on board a hydrographic sounding ship at Trois-Rivières, on the St. Lawrence River, 130km upstream (southwest) of Québec City, on 22 October 1998 and simultaneously one reference station in the Canadian Coast Guard (CCG) DGPS and OTF network (Figure 1 and 2).

Figure 1: GPS-equipped hydrographic sounding ship.



In the Kalman filter approach, if cycle slips have been removed correctly, we can expect the state parameter estimates to be unbiased and while they can be contaminated by the carrier-phase observation noise, the contamination is acceptable for the smoothing process.

Although the initial constant bias exists in the double-difference multipath estimates as shown in Figure 3, we can still see the divergence in the carrier-smoothed code pseudoranges. If the initial constant bias is eliminated, the only change that we can see is shifting of the divergence graph in Figure 4 along the y-axis. Therefore, no matter what the initial constant bias is, we can investigate the multipath divergence problem in a data set. By estimating the multipath in the Kalman filter, we can produce significantly better smoothed code pseudoranges.

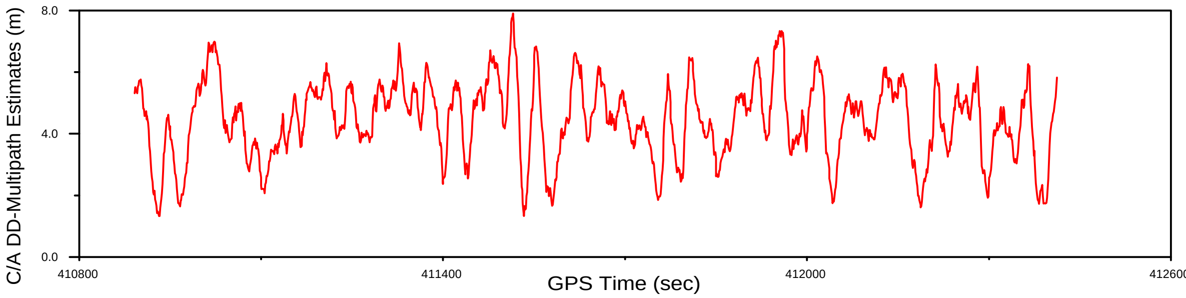


Figure 3: C/A-code double-difference multipath estimates for PRN23 and PRN21 pair.

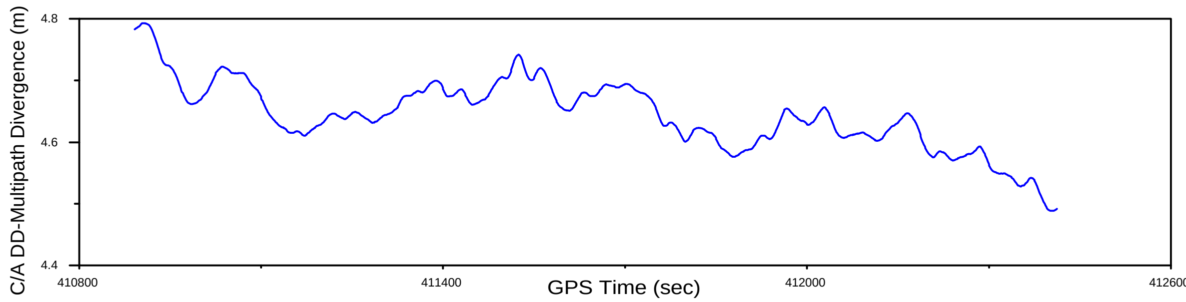


Figure 4: C/A-code double-difference multipath divergence for PRN23 and PRN21 pair.

Concluding Remarks

We have introduced the multipath divergence problem in the carrier-smoothed code pseudorange. In some applications, we cannot avoid this situation and the resulting smoothed pseudorange can diverge. To overcome this problem, we have augmented a Kalman filter to estimate the double-difference multipath and efficiently remove it from the carrier-smoothed code pseudorange.

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