Revised Refractive Index Formulae and Their Effect in Zenith Delay Prediction and Estimation

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Outline

- Atmospheric refraction
- Refractivity
- Zenith atmospheric propagation delay
- Impact of refractivity computation
- Impact of saturation vapour pressure computation
- Conclusions

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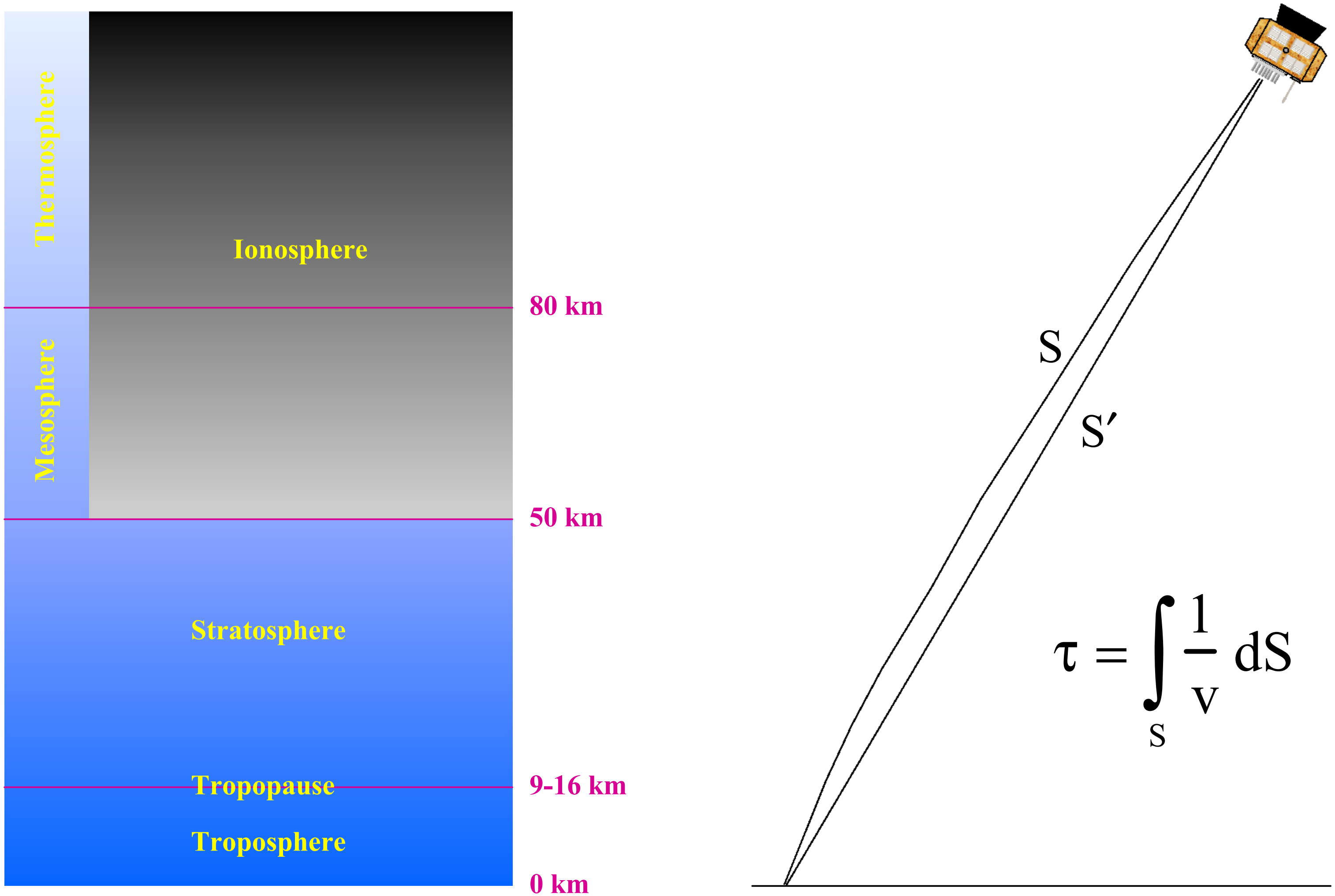
Atmospheric Refraction



$$\tau = \int_S \frac{1}{v}\, dS$$

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Refractive Index

- Refractive Index (*n*) – ratio of the speed of propagation of an EM wave in a vacuum (*c*) to the phase speed of propagation in a given medium (*v*)
- *v* less than *c* ⇒ excess path delay
- *n* variable throughout the atmosphere ⇒ ray bending
- Refractivity (*N*): $N = (n-1)\times 10^6$

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Refractivity - I

$$N = K_1' \frac{P_d^*}{T} + K_2 \frac{e}{T} + K_3 \frac{e}{T^2} + K_4 \frac{P_c}{T}$$

K_i	refractivity constants (determined in a laboratory)
P_d^*	partial pressure due to dry air (CO_2-free)
P_d	partial pressure due to dry air (including CO_2)
e	partial pressure due to water vapour
P_c	partial pressure due to CO_2
T	temperature

$$N = K_1 \frac{P_d}{T} + K_2 \frac{e}{T} + K_3 \frac{e}{T^2}$$

Refractivity - II

$$N = K_1\left(\frac{P_d}{T}\right)Z_d^{-1} + \left[K_2\frac{e}{T} + K_3\frac{e}{T^2}\right]Z_w^{-1}$$

dry wet

$$N = K_1 R_d \rho + \left[K_2'\frac{e}{T} + K_3\frac{e}{T^2}\right]Z_w^{-1}$$

hydrostatic non-hydrostatic ("wet")

Refractivity - III

Z_d^{-1} inverse compressibility factor for dry air

Z_w^{-1} inverse compressibility factor for water vapor

$$K_2^{'} = K_2 - K_1 \left(\frac{R_d}{R_w} \right)$$

R_d – specific gas constant for dry air ($R_d = 287.0586 \pm 0.0055$ J kg^{-1} K^{-1})

R_w – specific gas constant for water vapor ($R_w = 461.525 \pm 0.013$ J kg^{-1} K^{-1})

ρ – density for dry air

Neutral Atmosphere Delay

$$d_{na} = d_h^z \cdot m_h(\varepsilon) + d_{nh}^z \cdot m_{nh}(\varepsilon)$$

Zenith hydrostatic delay

Hydrostatic mapping function

Zenith non-hydrostatic ("wet") delay

Non-hydrostatic ("wet") mapping function

Zenith Propagation Delay

$$d_{na}^{z} = \int_{r_s}^{r_a} (n-1)\ dz$$

$$d_{na}^{z} = 10^{-6} \int_{r_s}^{r_a} N\ dz$$

$$d_{na}^{z} = 10^{-6} \int_{r_s}^{r_a} K_1 R_d \rho\ dz + 10^{-6} \int_{r_s}^{r_a} \left[K_2^{'} \frac{e}{T} + K_3 \frac{e}{T^2} \right] Z_w^{-1}\ dz$$

zenith hydrostatic delay (ZHD)

zenith non-hydrostatic delay (ZNhD)

Zenith Hydrostatic Delay

$$d_h^z = 10^{-6} \int_{r_s}^{r_a} K_1 R_d \rho \, dz$$

$$\int_{r_s}^{r_a} \rho \, dz = \int_{P_s}^{0} -\frac{dP}{g} = \frac{P_s}{g_m}$$

$$\boxed{d_h^z = 10^{-6} K_1 R_d \frac{P_s}{g_m} = \zeta \frac{P_s}{f(\phi, H)}}$$

- ZHD can be determined accurately if surface pressure measurements are available
- ZHD depends on the choice of K_1 and g_m

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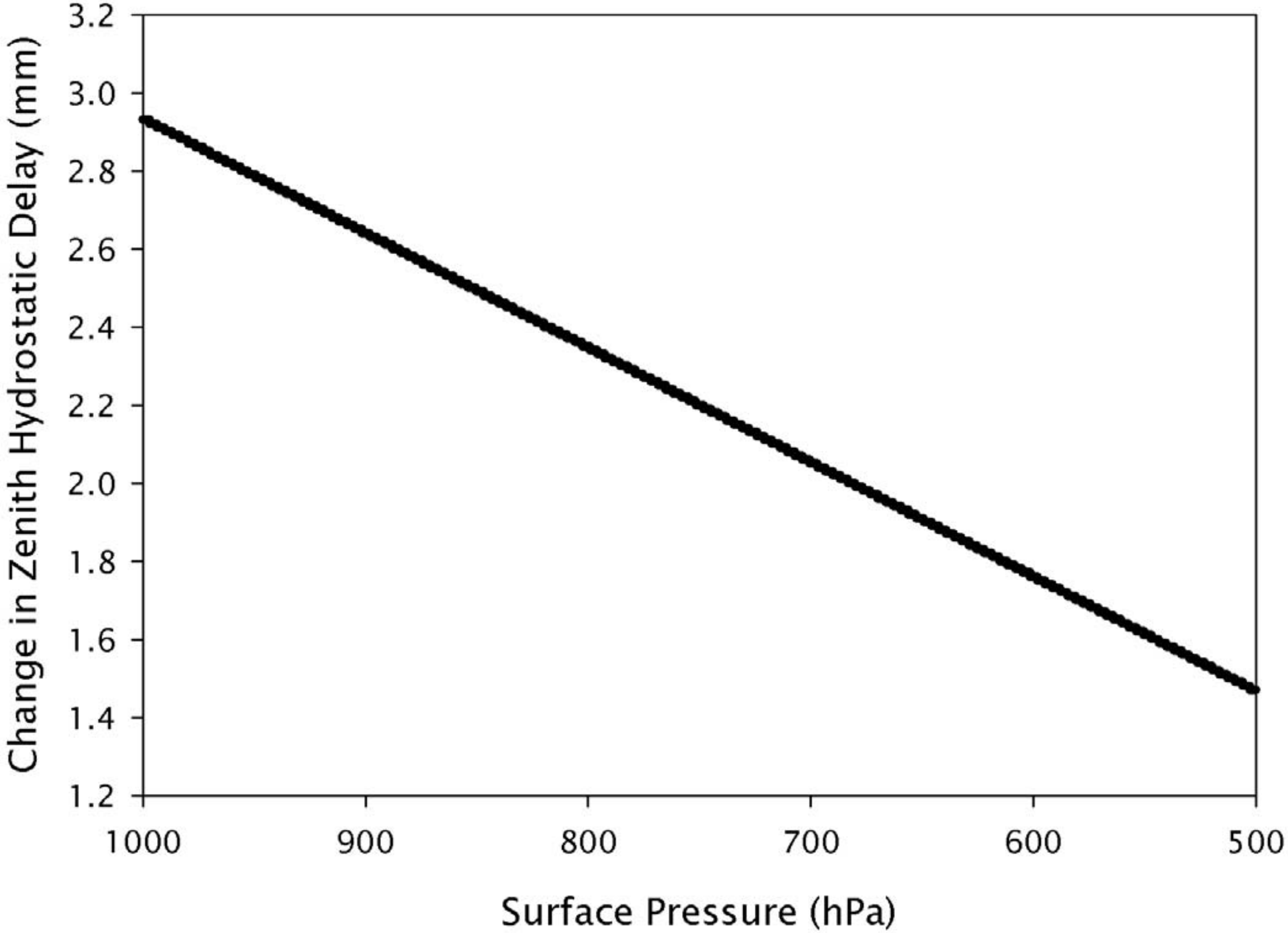
ZHD Changes for $\Delta K_1 = 0.1$ K hPa^{-1}



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Zenith Non-hydrostatic Delay

$$d^z_{nh} = 10^{-6} \int_{r_s}^{r_a} \left[K_2^{'} \frac{e}{T} + K_3 \frac{e}{T^2} \right] Z_w^{-1} \, dz$$

- e is variable in space and time
- Distribution of water vapour with altitude is not known
- There is no closed solution for the ZNhD
- Effect of changes in refractivity constants can be analyzed by raytracing profiles of e and T obtained by radiosondes

Refractivity Constants

Formula	K_1 (K hPa^{-1})	K_2 (K hPa^{-1})	K_3 (K^2 hPa^{-1})	K_2' (K hPa^{-1})	ζ (m hPa^{-1})
BOUD63	77.594 (±0.075)	71.968 (±10.5)	375406 (±3000)	23.7 (±10.5)	0.0022766 (±0.0000022)
IAG63	77.624 (–)	64.700 (–)	371897 (–)	16.4 (–)	0.0022775 (–)
LIEB77	77.676 (±0.023)	71.631 (–)	374656 (–)	23.3 (–)	0.0022790 (–)
LIEB96	77.640 (±0.023)	71.7 (–)	374670 (–)	23.4 (–)	0.0022779 (–)
BNB300 (300 ppm CO2)	77.691 (±0.013)	71.97 (±10.5)	375406 (±3000)	23.6 (±10.5)	0.0022794 (±0.0000004)
RUEG375 (375 ppm CO2)	77.6890 (±0.0094)	71.2952 (±1.3)	375463 (±760)	23.0 (±1.3)	0.0022794 (±0.0000003)

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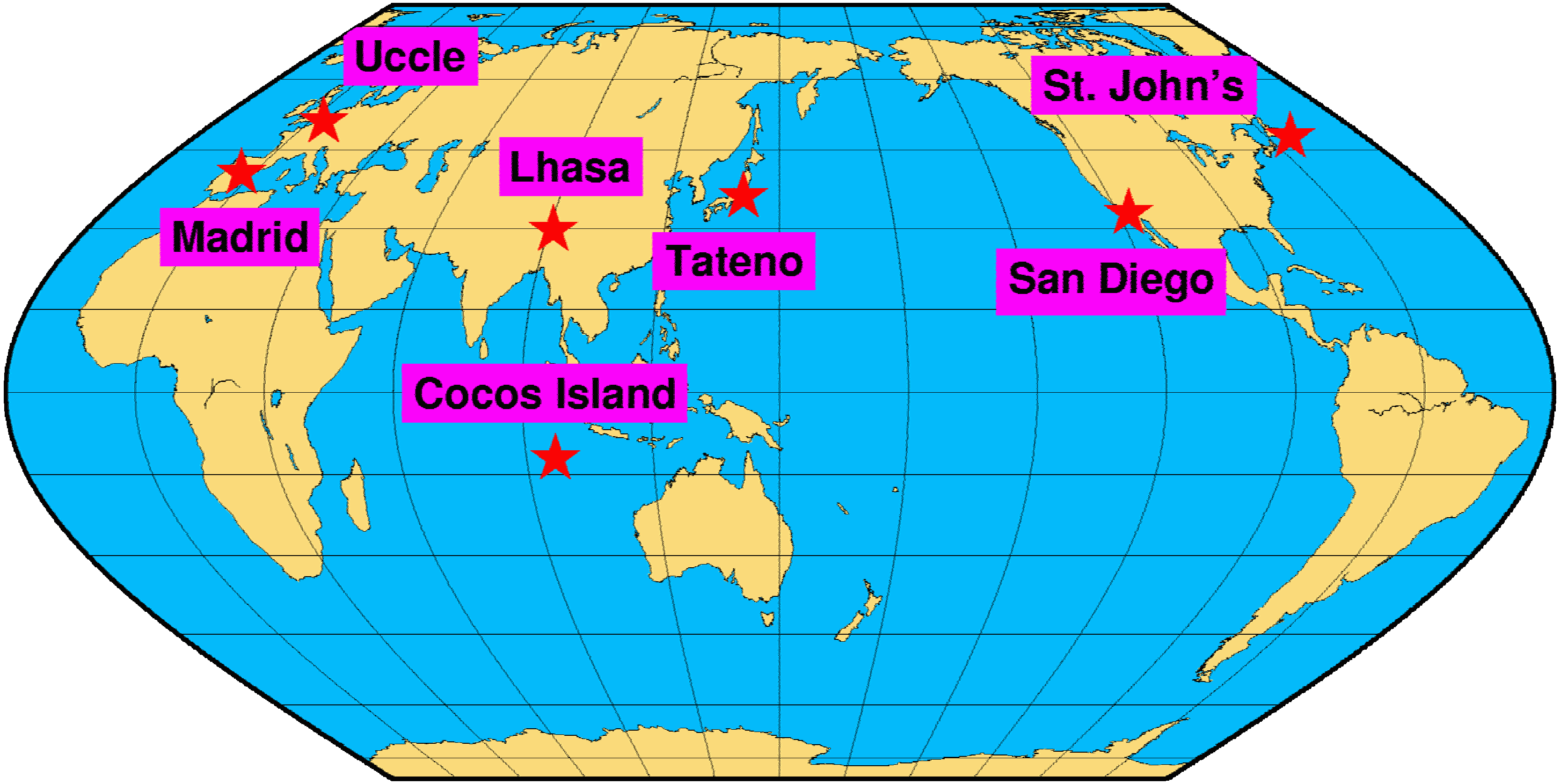
Radiosonde Locations



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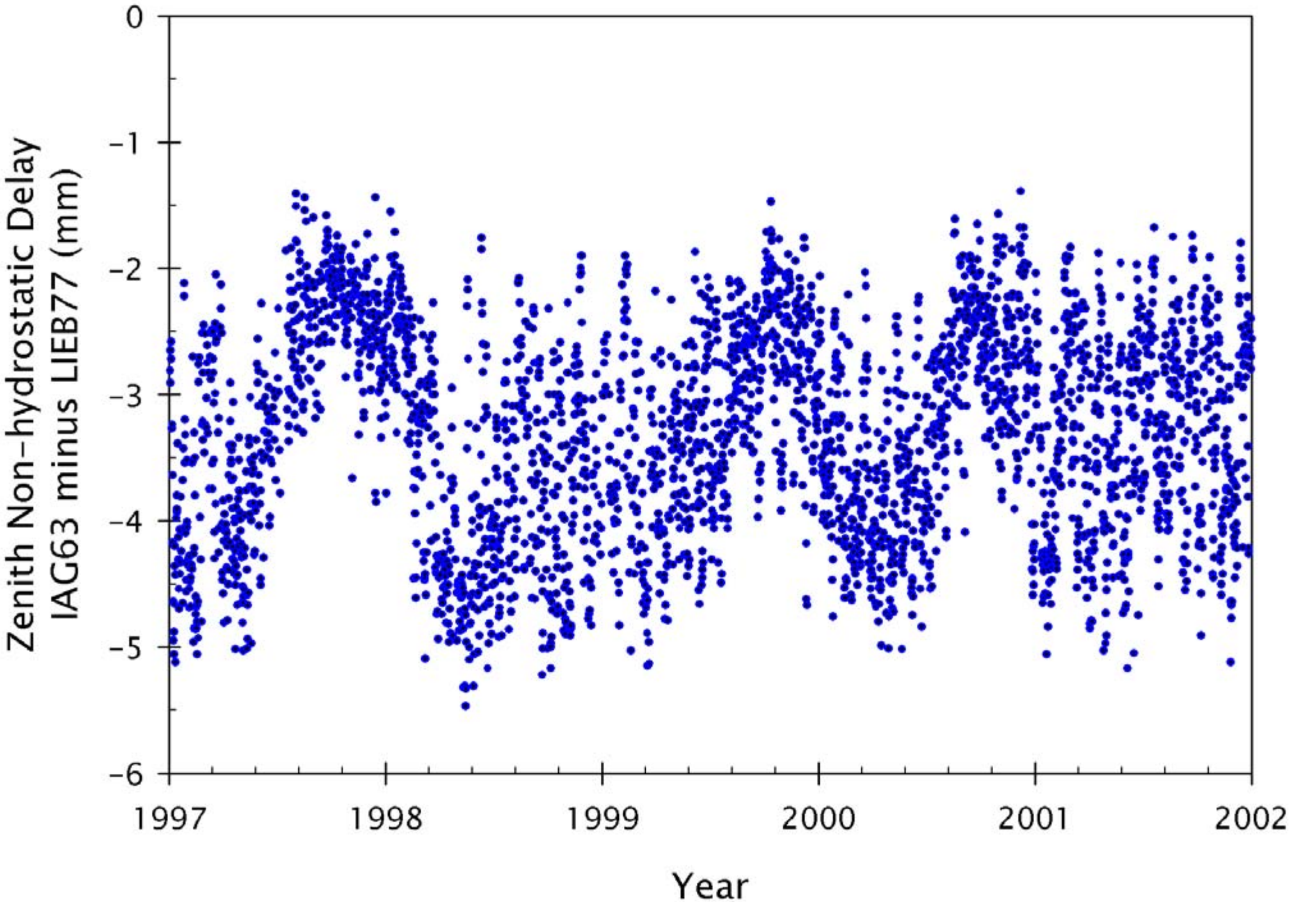
ZNhD: IAG63 minus LIEB77

Cocos Island



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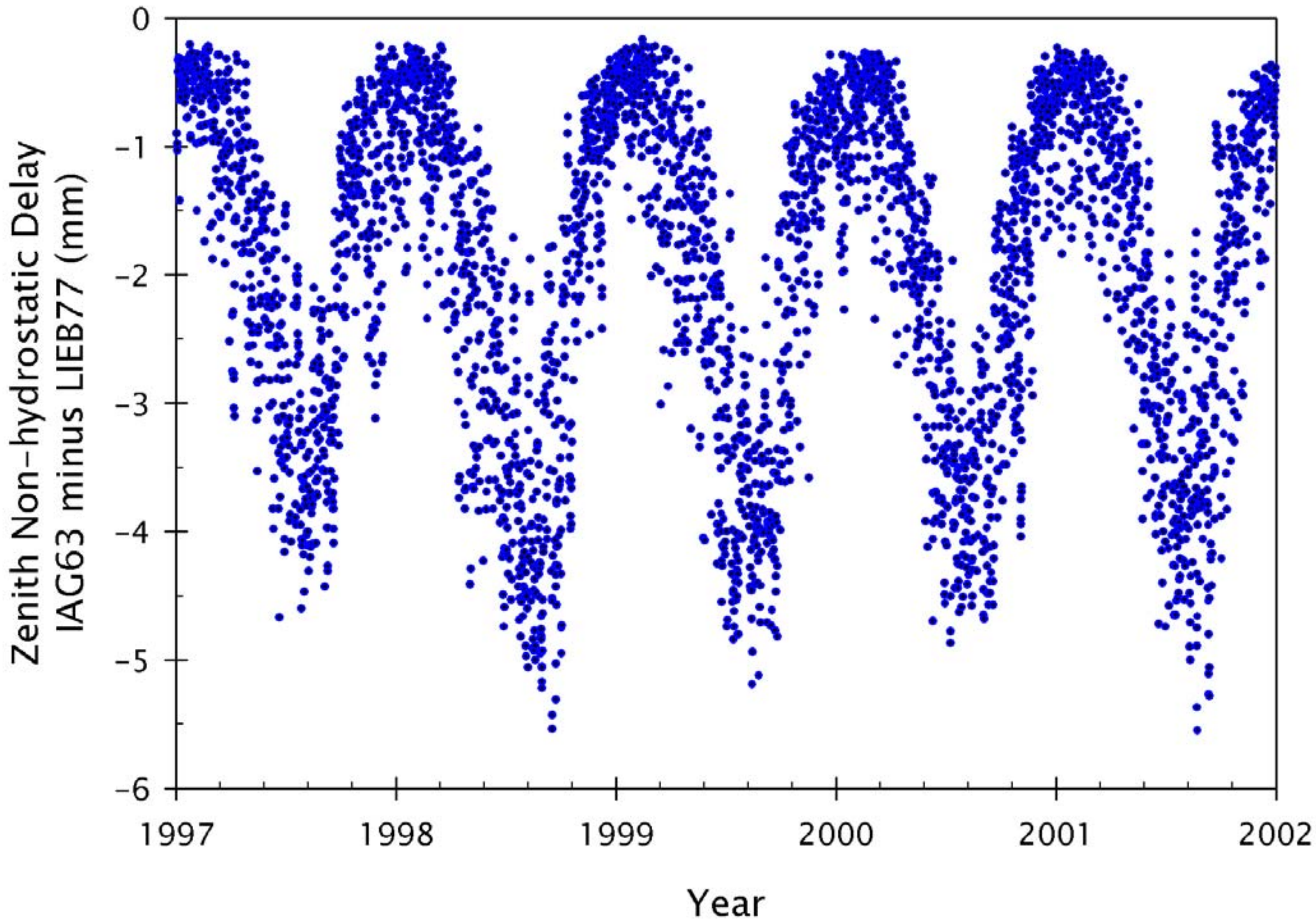
ZNhD: IAG63 minus LIEB77

Tateno



ZNhD: IAG63 minus LIEB77

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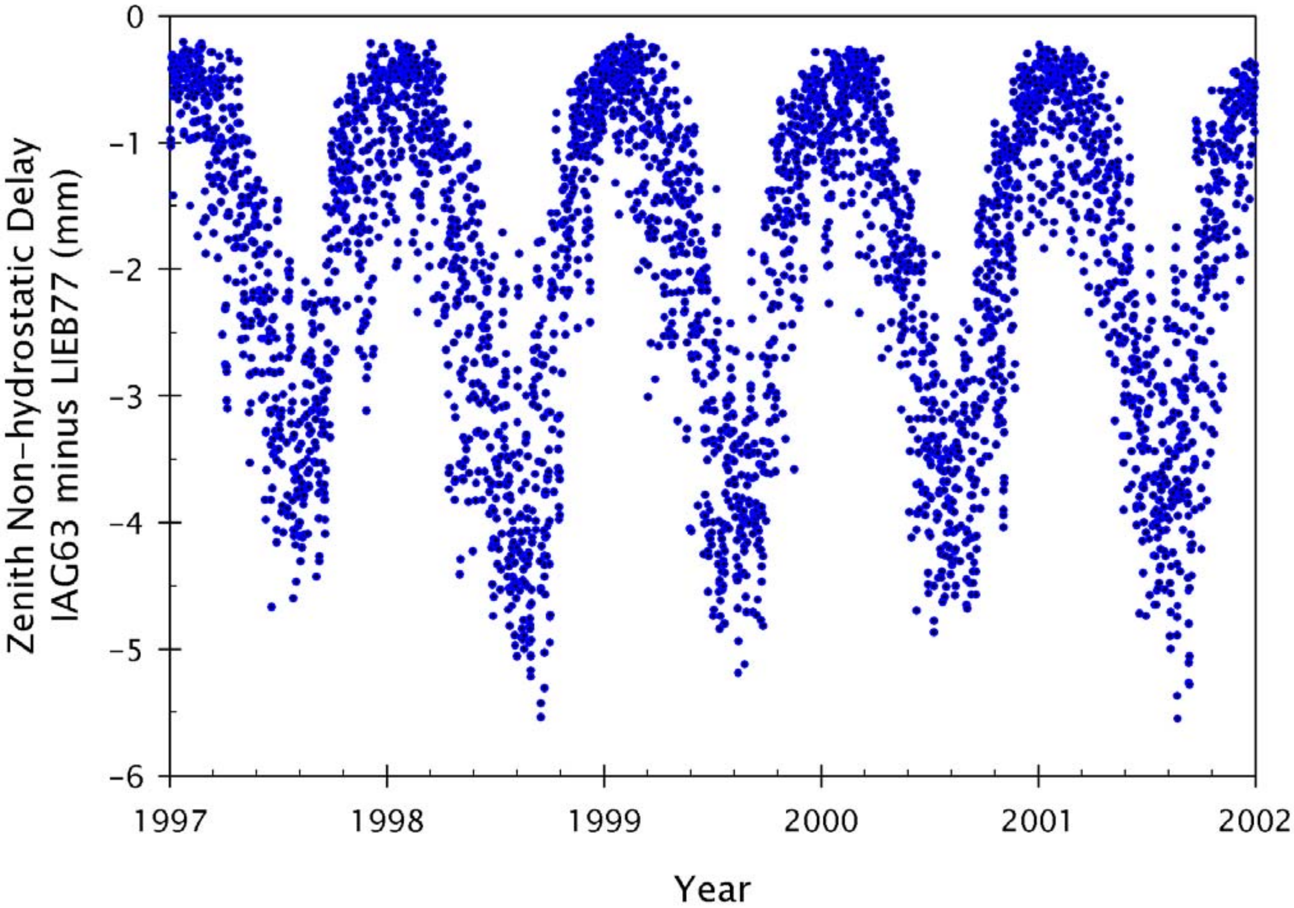
Lhasa



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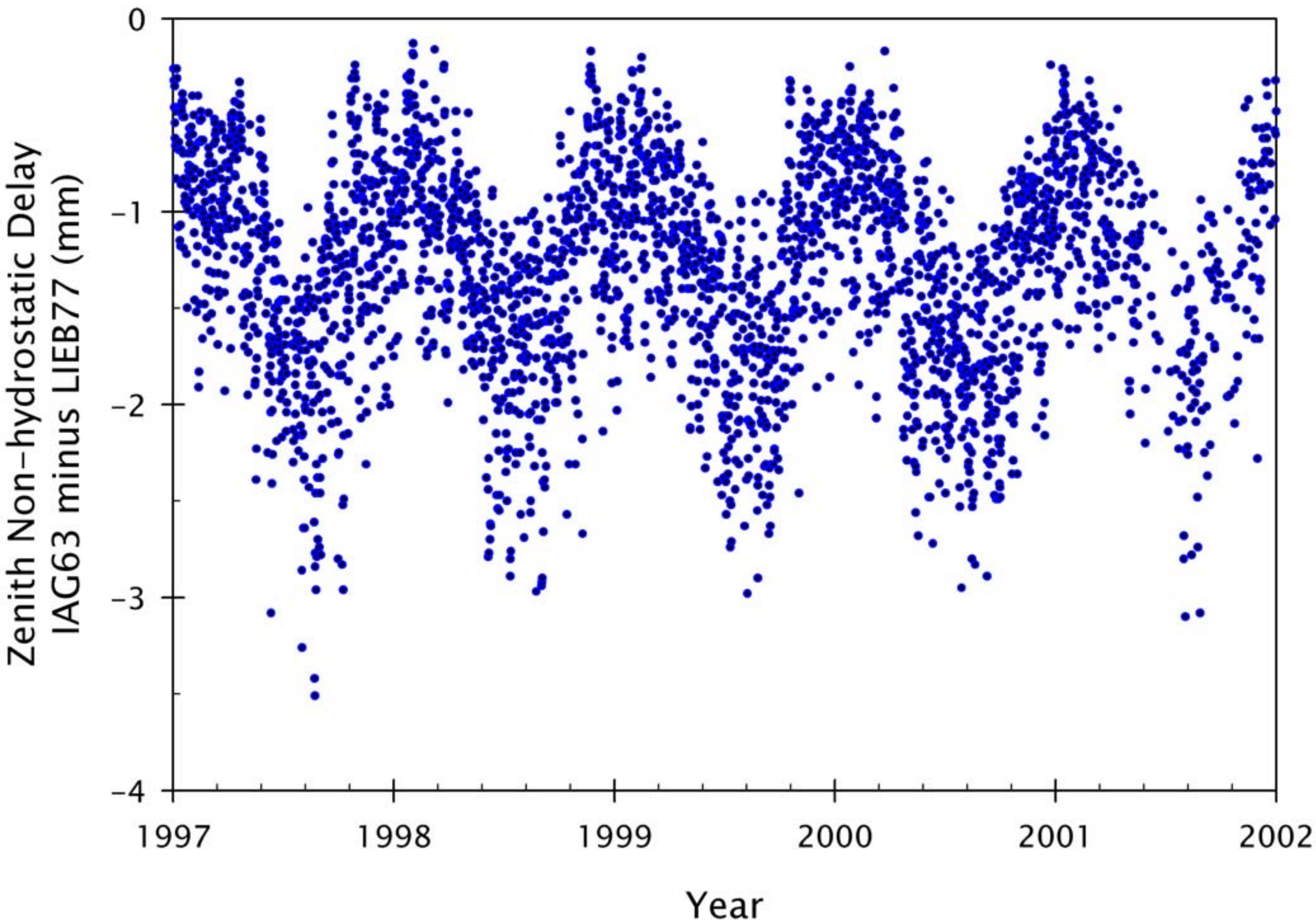
ZNhD: IAG63 minus LIEB77

Uccle



ZNhD: IAG63 minus LIEB77

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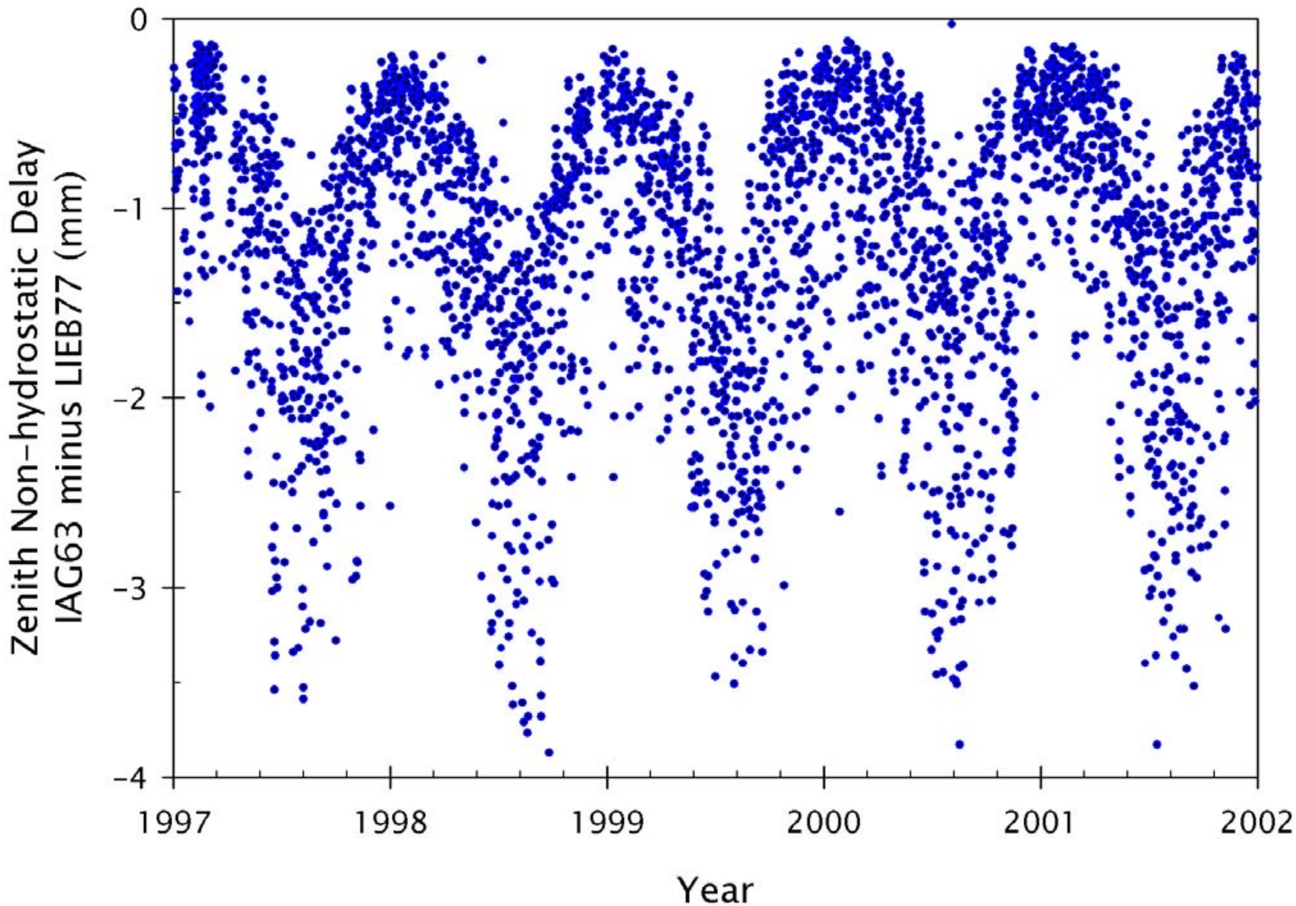
St. John's



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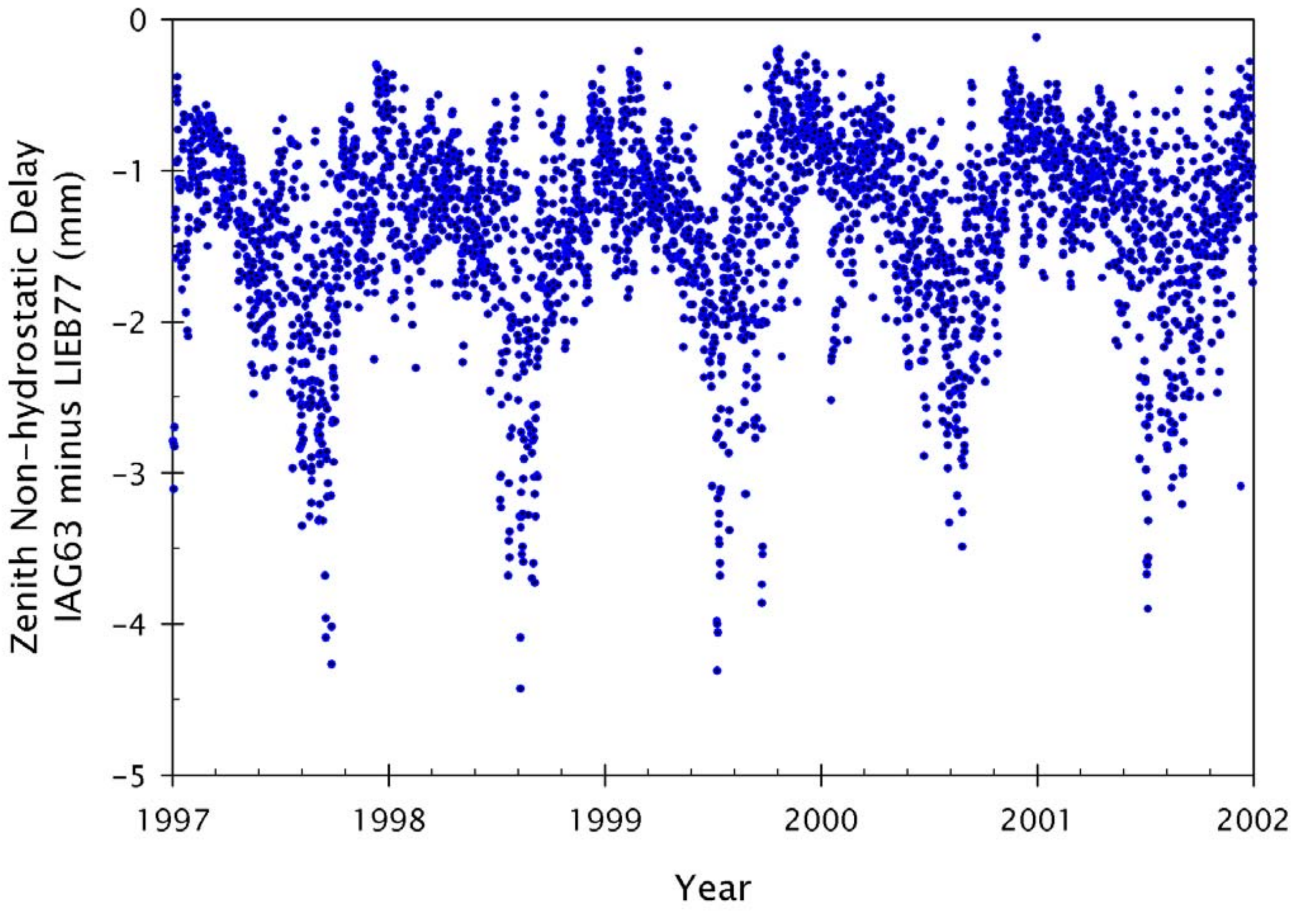
ZNhD: IAG63 minus LIEB77

San Diego



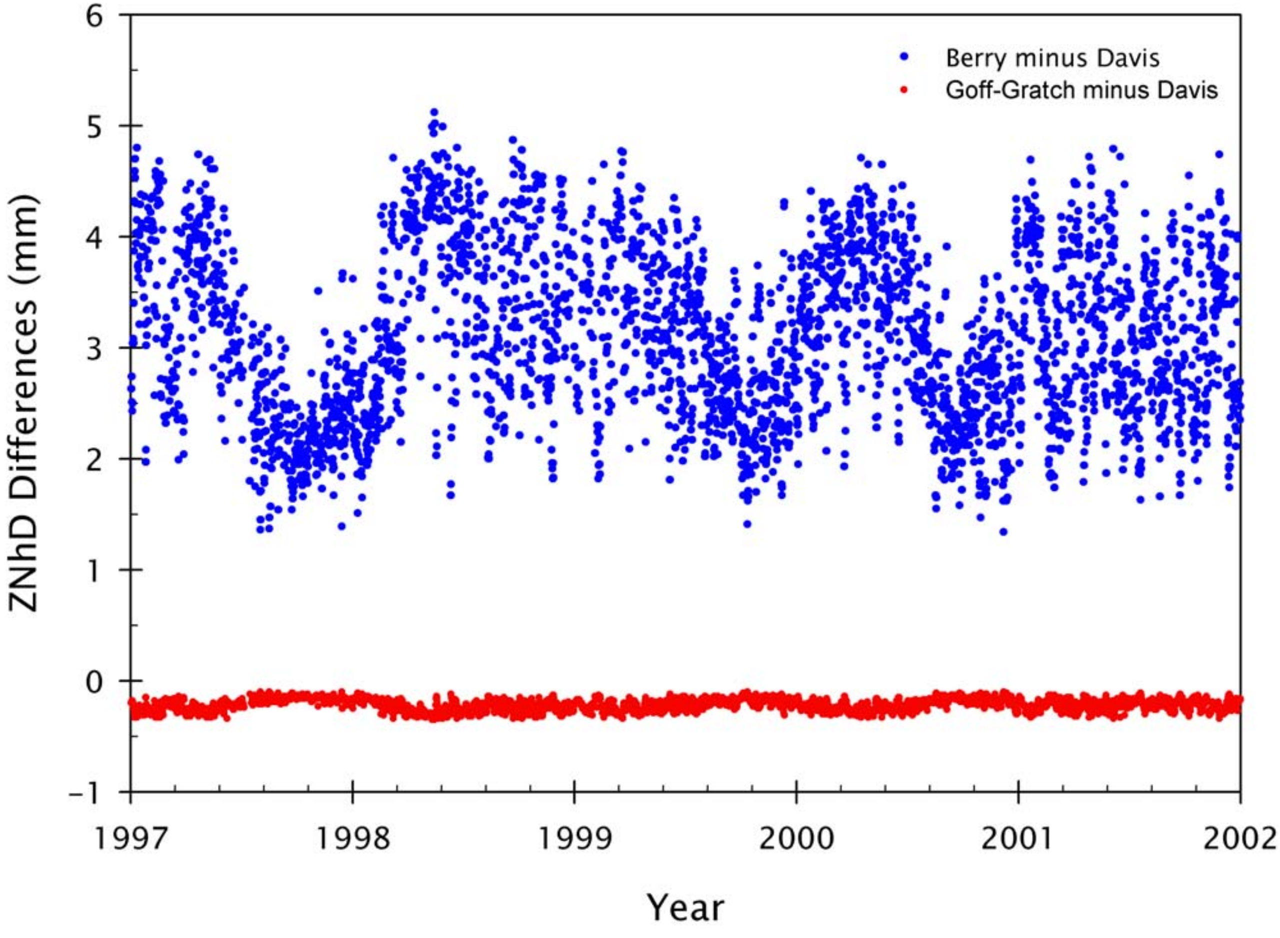
Saturation Vapour Pressure

- Water vapour pressure is computed from RH (or dew point) and saturation vapour pressure (e_{sw})
- Different formulae for saturation vapour pressure available (Berry, Goff-Gratch, Wexler, Davis, etc.)
- Differences can amount to a few mm in ZNhD computation
- e_{sw} of moist air $\neq$ e_{sw} of pure water

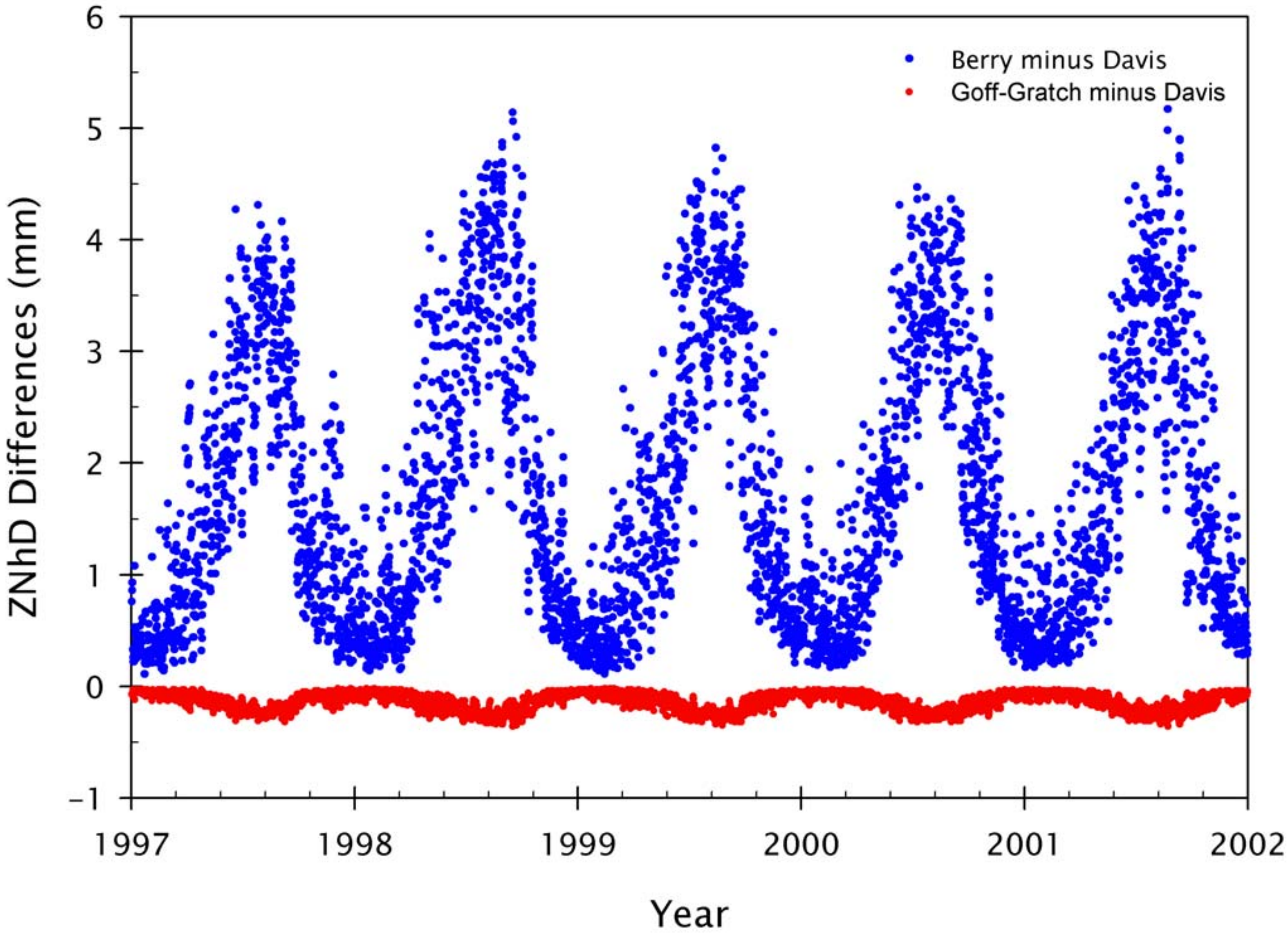
  $\Rightarrow$ Use of enhancement factor (differences $<$ 1 mm)

e_{sw} Computation

Cocos Island



Tateno



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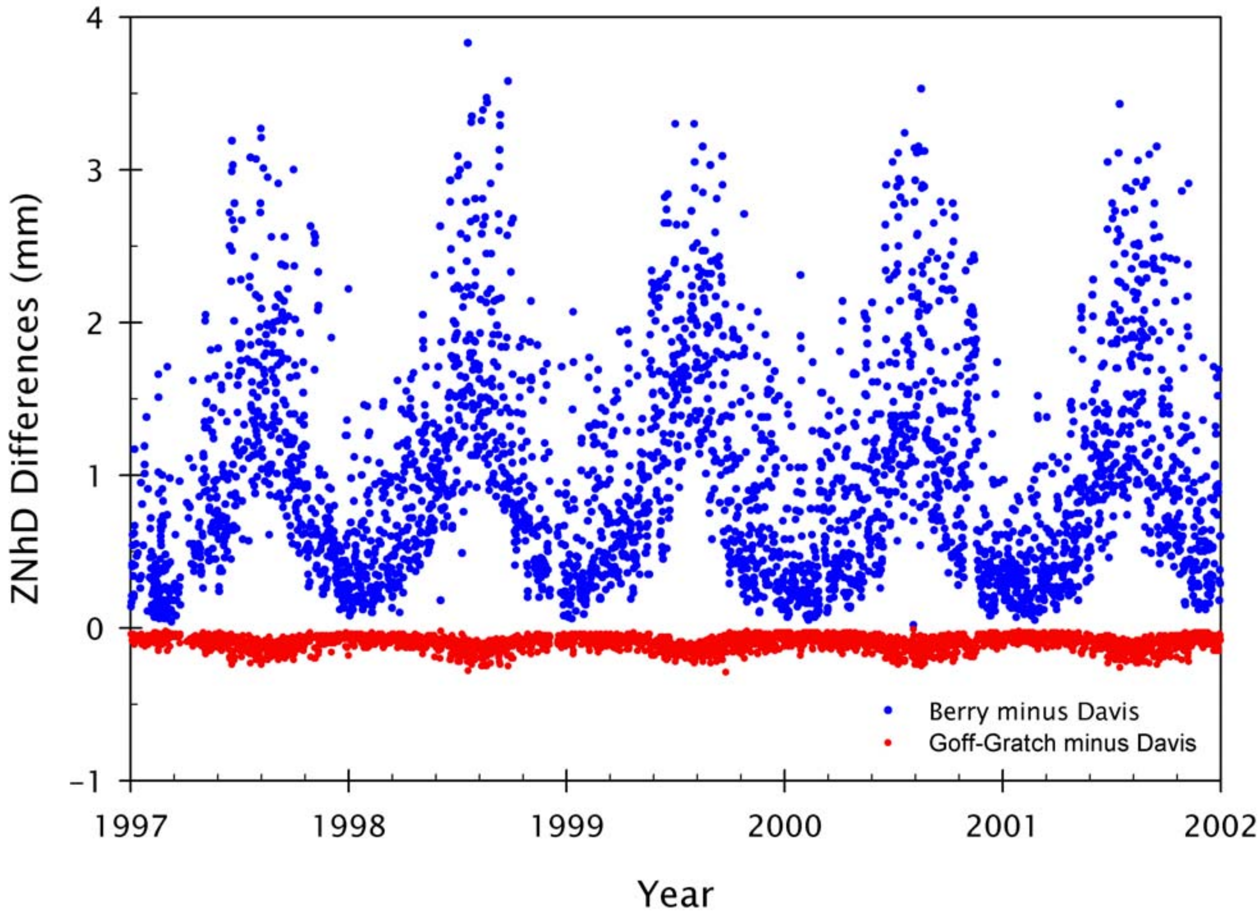
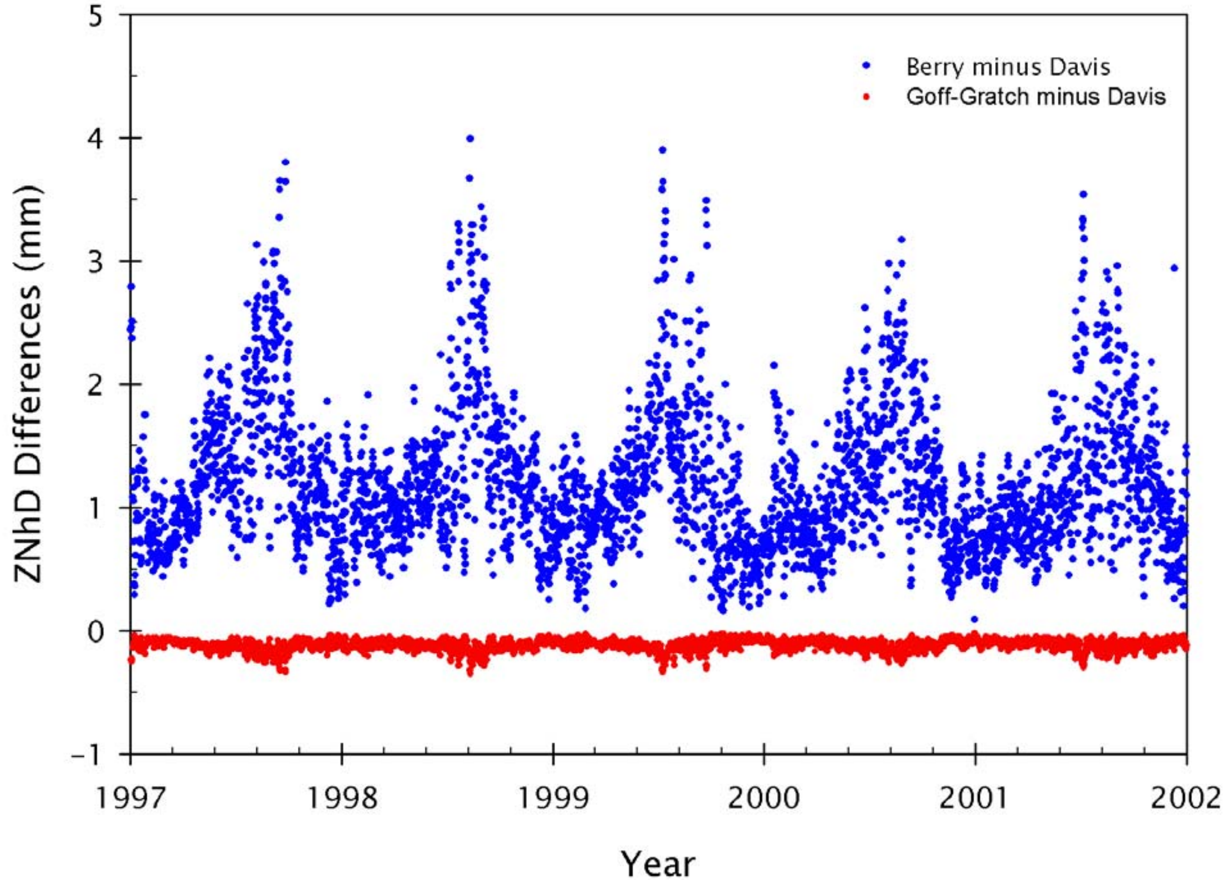

ZNhD Differences (mm)
4
3
2
1
0
−1
1997
1998
1999
2000
2001
2002
Year
Berry minus Davis
Goff-Gratch minus Davis

e_{sw} Computation

San Diego



Concluding Remarks

- A new formula for refractivity computation will have impact in modeling the neutral-atmosphere propagation delay in high-precision applications
- Models for ZHD prediction, such as Saastamoinen, can be easily reformulated to reflect any future recommendation
- Effects in existing models for ZNhD prediction are not significant (below the prediction accuracy)
- A recommendation for the computation of saturation vapour pressure is highly desirable