Revised Refractive Index Formulae and Their Effect in Zenith Delay Prediction and Estimation

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Outline

• Atmospheric refraction
• Refractivity
• Zenith atmospheric propagation delay
• Impact of refractivity computation
• Impact of saturation vapour pressure computation
• Conclusions
Atmospheric Refraction

$$\tau = \int_{s}^{S} \frac{1}{v} dS$$

- Ionosphere
- Stratosphere
- Tropopause
- Troposphere
- Thermosphere

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Refractive Index

- Refractive Index \((n)\) – ratio of the speed of propagation of an EM wave in a vacuum \((c)\) to the phase speed of propagation in a given medium \((v)\)
- \(v\) less than \(c\) \(\Rightarrow\) excess path delay
- \(n\) variable throughout the atmosphere \(\Rightarrow\) ray bending
- Refractivity \((N)\): \[ N = (n - 1) \times 10^6 \]
Refractivity - I

\[ N = K_1 \frac{P_d^*}{T} + K_2 \frac{e}{T} + K_3 \frac{e}{T^2} + K_4 \frac{P_c}{T} \]

- \( K_i \) : refractivity constants (determined in a laboratory)
- \( P_d^* \) : partial pressure due to dry air (CO₂-free)
- \( P_d \) : partial pressure due to dry air (including CO₂)
- \( e \) : partial pressure due to water vapour
- \( P_c \) : partial pressure due to CO₂
- \( T \) : temperature
Refractivity - II

\[ N = K_1 \left( \frac{P_d}{T} \right) Z_d^{-1} + \left[ K_2 \frac{e}{T} + K_3 \frac{e}{T^2} \right] Z_w^{-1} \]

dry

\[ N = K_1 R_d \rho + \left[ K'_2 \frac{e}{T} + K_3 \frac{e}{T^2} \right] Z_w^{-1} \]

hydrostatic non-hydrostatic ("wet")
Refractivity - III

\[ Z_d^{-1} \text{ inverse compressibility factor for dry air} \]

\[ Z_w^{-1} \text{ inverse compressibility factor for water vapor} \]

\[ K'_2 = K_2 - K_1 \left( \frac{R_d}{R_w} \right) \]

\[ R_d \text{ – specific gas constant for dry air (} R_d = 287.0586 \pm 0.0055 \text{ J kg}^{-1} \text{ K}^{-1} \)\]

\[ R_w \text{ – specific gas constant for water vapor (} R_w = 461.525 \pm 0.013 \text{ J kg}^{-1} \text{ K}^{-1} \)\]

\[ \rho \text{ – density for dry air} \]
Neutral Atmosphere Delay

\[ d_{na} = d^z_h \cdot m_h(\varepsilon) + d^z_{nh} \cdot m_{nh}(\varepsilon) \]

Hydrostatic mapping function

- Zenith hydrostatic delay

Non-hydrostatic ("wet") mapping function

- Zenith non-hydrostatic ("wet") delay
Zenith Propagation Delay

$$d^z_{na} = \int_{z_s}^{r_a} (n - 1) \, dz$$

$$d^z_{na} = 10^{-6} \int_{z_s}^{r_a} N \, dz$$

$${d^z_{na}} = 10^{-6} \int_{z_s}^{r_a} K_1 R d\rho \, dz + 10^{-6} \int_{z_s}^{r_a} \left[ K_2 \frac{e}{T} + K_3 \frac{e}{T^2} \right] Z_w^{-1} \, dz$$

zenith hydrostatic delay (ZHD)

zenith non-hydrostatic delay (ZNhD)
Zenith Hydrostatic Delay

\[ d^z_h = 10^{-6} \int_{r_s}^{r_a} K_1 R_d \rho \, dz \]

\[ \int_{r_s}^{r_a} \rho \, dz = \int_{P_s}^{0} \frac{dP}{g} = \frac{P_s}{g_m} \]

\[
\begin{bmatrix}
\int_{r_s}^{r_a} \rho \, dz = \int_{P_s}^{0} \frac{dP}{g} = \frac{P_s}{g_m} \\
\end{bmatrix}
\]

\[ d^z_h = 10^{-6} K_1 R_d \frac{P_s}{g_m} = \zeta \frac{P_s}{f(\phi, H)} \]

- ZHD can be determined accurately if surface pressure measurements are available
- ZHD depends on the choice of \( K_1 \) and \( g_m \)
ZHD Changes for $\Delta K_1 = 0.1 \text{ K hPa}^{-1}$
Zenith Non-hydrostatic Delay

\[ d_{nh}^z = 10^{-6} \int_{r_s}^{r_a} \left[ K'_2 \frac{e}{T} + K_3 \frac{e}{T^2} \right] Z_w^{-1} \, dz \]

- \( e \) is variable in space and time
- Distribution of water vapour with altitude is not known
- There is no closed solution for the ZNhD
- Effect of changes in refractivity constants can be analyzed by raytracing profiles of \( e \) and \( T \) obtained by radiosondes
## Refractivity Constants

<table>
<thead>
<tr>
<th>Formula</th>
<th>( K_1 ) (K hPa(^{-1}))</th>
<th>( K_2 ) (K hPa(^{-1}))</th>
<th>( K_3 ) (K(^2) hPa(^{-1}))</th>
<th>( K_4 ) (K hPa(^{-1}))</th>
<th>( \zeta ) (m hPa(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOUD63</td>
<td>77.594 (±0.075)</td>
<td>71.968 (±10.5)</td>
<td>375406 (±3000)</td>
<td>23.7 (±10.5)</td>
<td>0.0022766 (±0.0000022)</td>
</tr>
<tr>
<td>IAG63</td>
<td>77.624 (–)</td>
<td>64.700 (–)</td>
<td>371897 (–)</td>
<td>16.4 (–)</td>
<td>0.0022775 (–)</td>
</tr>
<tr>
<td>LIEB77</td>
<td>77.676 (±0.023)</td>
<td>71.631 (–)</td>
<td>374656 (–)</td>
<td>23.3 (–)</td>
<td>0.0022790 (–)</td>
</tr>
<tr>
<td>LIEB96</td>
<td>77.640 (±0.023)</td>
<td>71.7 (–)</td>
<td>374670 (–)</td>
<td>23.4 (–)</td>
<td>0.0022779 (–)</td>
</tr>
<tr>
<td>BNB300 (300 ppm CO(_2))</td>
<td>77.691 (±0.013)</td>
<td>71.97 (±10.5)</td>
<td>375406 (±3000)</td>
<td>23.6 (±10.5)</td>
<td>0.0022794 (±0.0000004)</td>
</tr>
<tr>
<td>RUEG375 (375 ppm CO(_2))</td>
<td>77.6890 (±0.0094)</td>
<td>71.2952 (±1.3)</td>
<td>375463 (±760)</td>
<td>23.0 (±1.3)</td>
<td>0.0022794 (±0.0000003)</td>
</tr>
</tbody>
</table>
Radiosonde Locations

- Uccle
- Madrid
- Lhasa
- Tatenos
- Cocos Island
- St. John’s
- San Diego
ZNhD: IAG63 minus LIEB77

Cocos Island

Year

Year

Zenith Non-hydrostatic Delay

IAG63 minus LIEB77 (mm)
ZNhD: IAG63 minus LIEB77

Tateno

![Graph showing ZN
hD: IAG63 minus LIEB77 for Tateno over the years 1997 to 2002, with data points distributed unevenly across the years.]
ZNhD: IAG63 minus LIEB77

Lhasa
ZNhD: IAG63 minus LIEB77

Uccle

Year

Zenith Non-hydrostatic Delay
IAG63 minus LIEB77 (mm)

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M&L, 2002
ZNhD: IAG63 minus LIEB77

St. John’s
ZNhD: IAG63 minus LIEB77

San Diego

Zenith Non-hydrostatic Delay
IAG63 minus LIEB77 (mm)

Year

Saturation Vapour Pressure

- Water vapour pressure is computed from RH (or dew point) and saturation vapour pressure ($e_{sw}$).
- Different formulae for saturation vapour pressure available (Berry, Goff-Gratch, Wexler, Davis, etc.).
- Differences can amount to a few mm in ZNhD computation.
- $e_{sw}$ of moist air $\neq e_{sw}$ of pure water.
  \[ \Rightarrow \text{Use of enhancement factor (differences < 1 mm)} \]
e_{sw} Computation

Cocos Island

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$e_{SW}$ Computation

Tateno

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$e_{SW}$ Computation

St. John’s

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M&L, 2002
e_{sw} Computation

San Diego

![Graph showing ZNhD Differences (mm) over the years from 1997 to 2002. The graph includes data points for Berry minus Davis and Goff-Gratch minus Davis, with fluctuations seen throughout the years.]
Concluding Remarks

- A new formula for refractivity computation will have impact in modeling the neutral-atmosphere propagation delay in high-precision applications
- Models for ZHD prediction, such as Saastamoinen, can be easily reformulated to reflect any future recommendation
- Effects in existing models for ZNhD prediction are not significant (below the prediction accuracy)
- A recommendation for the computation of saturation vapour pressure is highly desirable