# The residual tropospheric propagation delay: How bad can it get?*

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## BIOGRAPHIES

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## ABSTRACT

The residual range delay error due to an incompletely modelled tropospheric propagation delay can usually be ignored by the average user of wide-area differential GPS. In general the atmosphere is "well behaved" and the use of standard range delay algorithms supplied with standard atmosphere parameter values will adequately model the "average" atmospheric conditions.

It is possible however, for the atmosphere to exhibit unusual conditions when the vertical profiles of total pressure or water vapour pressure are significantly different from average profiles. Extreme surface pressure differences on the order of 80 millibars from the global average have been recorded in the past. Conditions such as these could cause an unmodelled range error on the order of 2 metres at an elevation angle of five degrees. Of greater concern are high concentrations of water vapour producing zenith wet delays of up to half a metre. Such a condition could cause an unmodelled range error on the order of 3 to 4 metres at five degrees elevation angle. Therefore, it is theoretically possible that position-critical users of wide area differential GPS, relying on models using "average" atmospheric conditions, could have their vertical position accuracy degraded by up to several metres, depending on the satellite geometry.

The key question for such users is therefore: what is the frequency and magnitude of such conditions? Our research shows that significant extreme range errors are rare, provided that a good tropospheric delay model is used. We found that zenith delay errors greater than ±20 cm occurred only on the order of 7 in 100,000 cases sampled from across North America. The impact on position computation is determined by the elevation angle of the lowest satellite and whether or not a suitable weighting technique is used.

## INTRODUCTION

The residual tropospheric delay in GPS position estimation is often treated in a very off-hand manner — the assumption being that the effect is small, or that a simple estimation technique will take care of it. But what is the true impact? Just how much does the lower, electrically neutral, atmosphere vary in terms of refractivity and its effect on GPS applications? The aim of this paper is to provide answers to these questions and offer a quantification of the neutral atmospheric effect for wide-area differential users of GPS who are seeking metre level accuracy or better, and who may be operating within a "position critical" environment.

### *The Tropospheric Delay*

An electromagnetic signal propagating through the neutral atmosphere is affected by the constituent gases. The fact that their combined refractive index is slightly greater than unity gives rise to a decrease in the signal's velocity. This increases the time taken for the signal to reach a GPS receiver's antenna, increasing the equivalent path length (both effects are often referred to as the

* Presented at ION GPS-98, 11th International Technical Meeting of the Satellite Division of the Institute of Navigation, Nashville, Tennessee, 15-18 September, 1998.

"delay"). Refraction also bends the raypath and thereby lengthens it, further increasing the delay. Because the bulk of the delay occurs within the troposphere, the whole delay is often referred to solely as the "tropospheric delay".

By assuming that the neutral atmosphere is both horizontally stratified and azimuthally symmetric, the tropospheric delay can be modelled in two parts: the delay experienced in the zenith direction and the scaling of that delay to the delay experienced at the zenith angle of the raypath. The functions that undertake the scaling are usually termed mapping functions, although obliquity factor is sometimes used. This leads to the common formulation of zenith delays and mapping functions seen in the space geodetic and navigation literature. The formulation of the tropospheric delay can be described as:

$$d_{trop} = d_{hyd}^{z} \cdot m_{hyd} + d_{wet}^{z} \cdot m_{wet} \quad ,$$

where the total delay $d_{trop}$, is a function of the delays in the zenith direction caused by the atmospheric gases in hydrostatic equilibrium and by those gases not in hydrostatic equilibrium (primarily water vapour), $d_{hyd}^{z}$ and $d_{wet}^{z}$ respectively; and their mapping functions, $m_{hyd}$ and $m_{wet}$ respectively. The mapping functions are usually described as functions of the satellite elevation angle – the complement of the zenith angle.

The assumptions of horizontal stratification and azimuthal symmetry preclude the existence of gradients in the atmosphere. This is obviously not true and more sophisticated models have been derived to try and take into account the first-order effect of atmospheric gradients caused by pressure slopes and passing weather fronts (e.g. *Coster et al.* [1997]; *Gregorius and Blewitt* [1998]). These effects are estimated to be only at the decimetre level, at most, at low elevation angles. Errors from ignoring gradients will generally map into the horizontal position bias.

### *Tropospheric Delay Models*

When processing GPS observations, a value for the tropospheric delay is predicted from empirical models often using real-time values of the ambient temperature, pressure and water vapour pressure. Unfortunately, even with accurate real-time measurements, the true total delay can rarely be predicted with a degree of accuracy much better than a few percent. In theory, the hydrostatic component of the delay can be predicted in the zenith direction to the millimetre level, however the highly variable nature of atmospheric water vapour means that the accuracy of the wet delay is at the centimetre, or even decimetre level.

The reason for this behaviour is that the hydrostatic delay in the zenith direction is a function of the total surface pressure only, which, under conditions of hydrostatic equilibrium, represents the total weight of the column of air above the user. Analogously, the zenith wet delay is a function of the total precipitable water — the amount of vapour present in the column of air above the user. All tropospheric delay models represent these two parameters in various ways. The most common method for the wet delay is through a combination of surface parameters (temperature and water vapour pressure or relative humidity), and some kind of water vapour lapse rate (often known as the "lambda" parameter, see e.g., *Collins and Langley* [1997] and *Ifadis* [1993]). Not all tropospheric wet delay models are explicitly parameterised with such a parameter, but they often can be.

The problem with modeling the wet delay in this way is that unlike the hydrostatic delay, no simple physical law governs the distribution of water vapour in the lower atmosphere and hence a precise definition and evaluation of the lambda parameter is not possible. As a consequence the only way to accurately measure the lambda parameter is from some technique that attempts to sample the whole atmospheric column, such as a radiosonde or a radiometer. As these are impractical for real-time GPS users (and indeed for most other GPS users), the lambda parameter can only be represented empirically and consequently will always be associated with some error in the determination of the wet zenith delay.

Our previous tropospheric delay work at UNB has specified a composite tropospheric delay model (denoted as UNB3) to be used in aircraft receivers of the WAAS-GPS navigation system, currently being implemented across North America. The original definition of the model is based on the zenith delay algorithms of *Saastamoinen* [1973], the mapping functions of *Niell* [1996] and a table of atmospheric parameters derived from the U.S. 1966 Standard Atmosphere Supplements (more details in *Collins and Langley* [1997]).

A subsequent proposal has been made to replace the Niell mapping functions with the combined Black and Eisner function [*Black and Eisner*, 1984]), in the interests of computational simplicity. This change has a negligible impact on most of the results presented here, because we deal mainly with the residual portion of the zenith delay. The position error simulations described later in this paper were performed using the Black and Eisner mapping function, however the results are not expected to be significantly different when using the Niell functions.

The kernel of the UNB3 model is a look-up table of five atmospheric parameters that vary with respect to latitude and day-of-year. Linear interpolation is applied between latitudes and a sinusoidal function of the day-of-year attempts to model the seasonal variation. The parameters are total pressure, temperature and water vapour pressure at mean-sea-level, and two lapse rate parameters for temperature and water vapour. The lapse rates are used to scale the pressures and temperature to the user's altitude.

It is possible to replace the tabular values of the sea-level parameters with measured values to try and improve the model accuracy and reduce its susceptibility to extreme conditions. In general the lapse rate parameters can not be updated in real-time and therefore contribute the greatest inaccuracy. One aim of this paper is to examine the operation of the UNB3 model in two modes: as a wholly stand-alone model, denoted as "UNB3", and driven with surface measurements, denoted as "UNB3(SfcMet)".

The UNB3 model is designed to be an improvement over "first-generation" tropospheric delay models intended for navigational use, such as the Altshuler model [*Altshuler and Kalaghan*, 1974] and simple "constant value" models such as the NATO model [*NATO*, 1993] or the STI model [*Braash*, 1990; *Mendes and Langley*, 1998]. These latter models essentially use constant values of the parameters at mean-sea-level across the whole latitude range. Only the vertical variation is modelled, to represent the change in user's height (in an aircraft, for example). Models of this type are represented in this paper by the UNB1 model, which uses U.S. Standard Atmosphere parameter values of 1013.25 mbar (total pressure), 288.15 K (temperature), 11.7 mbar (water vapour pressure), 6.5 K/km (temperature lapse rate) and 3 (lambda parameter). (More details in *Collins and Langley* [1997].)

## METHODOLOGY

To provide the benchmark data for our investigation of propagation delay extremes, we ray-traced a portion of a 4 CD-ROM set of radiosonde data for North America that covers the years 1946-1996. The production of this data set was undertaken by the Forecast Systems Laboratory (FSL) of the National Oceanic and Atmospheric Administration (NOAA). The data consists of radiosonde soundings at mandatory and significant levels upto 100 mbar (~16 km) from almost all the radiosonde sites operating in the United States, Canada, Mexico, the Caribbean and Central America in the last fifty years. We have so far concentrated on the last ten full years of available data (1987-1996). This represents an average of 173 stations per year and approximately 1 million soundings in total.

Almost all atmospheric water vapour is found well below the 16 kilometre level and so the truncation of the CD-ROM data sets at this height pose no problem in this respect. For the purposes of ray-tracing the hydrostatic delay however, the temperature profile must be extended above this height. This can be done by using a suitable temperature profile. We have used the CIRA86 model [*Fleming et al.*, 1988] which provides monthly mean temperatures for every 5 kilometres of altitude upto 120 km at every 10 degrees of latitude. The required profile in time and location is produced by computing a weighted average of the four profiles surrounding a radiosonde launch site and then offsetting that profile to match the radiosonde temperature profile value at the truncation height.

We have undertaken a series of tests to ascertain whether the use of radiosonde profiles truncated above 100 mbar was acceptable. A small selection of profiles spread across North America and containing readings up to 10 or 20 mbar were obtained on-line from the University of Wyoming (http://www-das.uwyo.edu/upper air/sounding.html). Each profile was processed both in full and then after being truncated at 100 mbar. The largest delay discrepancy between the two profiles was on the order of 1 cm at five degrees elevation angle. This level of accuracy is easily sufficient for our purposes.

Other investigators (e.g. *Coster at al.* [1996]) have shown that ray-traced radiosonde profiles of the zenith delay are only accurate at the centimetre level compared to other instruments, such as radiometers. Because our truncated data limits the precision of the ray-trace data and because there are inherent limitations in recording and representing any water vapour profile with a radiosonde, the accuracy of the following results is only at the centimetre level.

## RESULTS

The zenith delay values computed by the various models were subtracted from the zenith ray-trace values to give the residual tropospheric delay and hence the model errors. The surface values used to drive UNB3(SfcMet) were taken from the radiosonde soundings, as were the location and time parameters (latitude, height, day-of-year) required by the models. The results are presented in three parts: the average statistical performance of the tropospheric delay models; the extreme delay errors; and the impact of those extremes on position computations simulated with real GPS ephemerides.

### *Average model performance*

To test the distributions of the residuals we can utilize Gaussian plots to compare them to a standardised Normal distribution. These are presented in Figure 1 and Figure 2. Figure 1 indicates how some tropospheric delay models can be biased by a large amount and skewed towards large residuals. The mean of the Altshuler model residuals is 15.9 cm with a standard deviation of 8.1 cm. Careful choice of parameter values, as was done for the UNB1 model, can provide a near zero-mean model, but the distribution is highly skewed and with a large standard deviation. The statistics for UNB1 are mean = 1.9 cm and standard deviation = 8.5 cm.

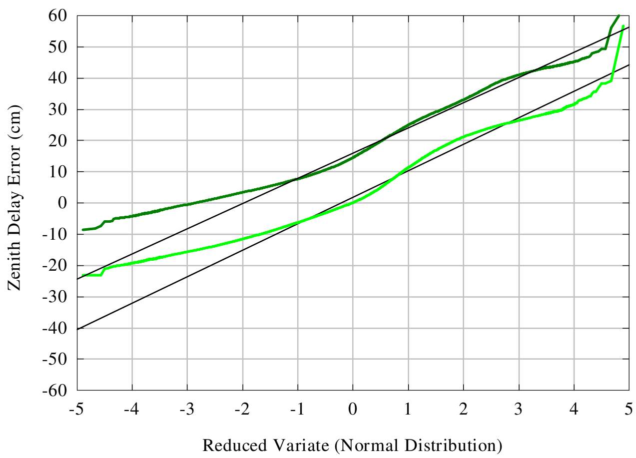


Figure 1. Gaussian plot of zenith delay residuals. (Dark line — Altshuler; light line — UNB1; thin lines — best fit Normal distributions.)

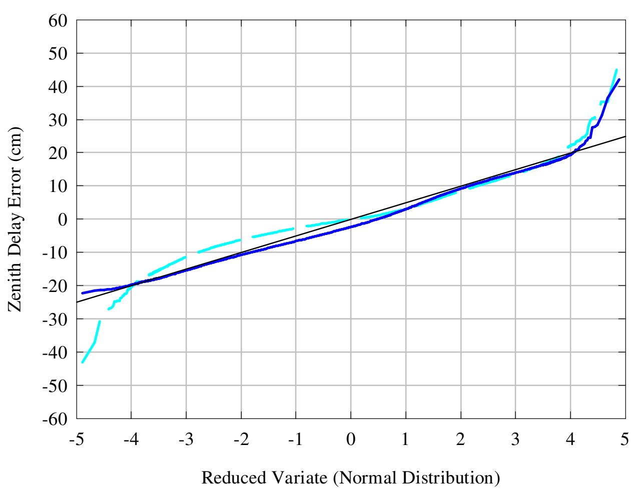


Figure 2. Gaussian plot of zenith delay residuals. (Solid line — UNB3; dashed line — UNB3(SfcMet); thin line — zero mean, 5 cm standard deviation Normal distribution.)

The distributions of residuals of both UNB3 models are shown in Figure 2 along with a theoretical distribution of zero mean and 5 cm standard deviation. It can be seen that this distribution characterises the residuals of both UNB3 models quite well upto approximately $\pm 4\sigma$ where the value of the residuals is almost exactly $\pm 20$ cm. Beyond the $4\sigma$ level, the lower bounds for UNB3 become progressively more conservative because the magnitude of the negative residuals appears to be leveling off. The residuals for UNB3(SfcMet) beyond the same point however are drastically underestimated. The residuals beyond the upper bound for both models are also drastically underestimated by a Normal distribution.

These plots indicate that characterising tropospheric delay errors using a Normal distribution beyond the $\pm 4\sigma$ level cannot be recommended, especially with simpler models, because the true distribution will be drastically underestimated. The probability level equivalent to $4\sigma$ in a Normal distribution is 99.994%, which is very high, however safety critical systems may demand even higher levels.

Table 1 shows the mean and standard deviation statistics for the two UNB3 models. The average performance of the two is very consistent from year-to-year, indicating that one year's-worth of data is sufficient to quantify the average, or typical, performance of a tropospheric delay model.

Table 1. Statistics for UNB3 and UNB3(SfcMet) models. (Units in centimetres.)

| Year | UNB3 | | UNB3(SfcMet) | |
|---|---|---|---|---|
| | Mean | S.D. | Mean | S.D. |
| 1987 | –1.7 | 5.0 | 0.3 | 3.3 |
| 1988 | –2.0 | 5.0 | 0.3 | 3.4 |
| 1989 | –1.9 | 4.8 | 0.2 | 3.4 |
| 1990 | –1.8 | 4.8 | 0.2 | 3.3 |
| 1991 | –1.6 | 4.9 | 0.4 | 3.3 |
| 1992 | –1.9 | 4.7 | 0.4 | 3.3 |
| 1993 | –1.7 | 4.9 | 0.4 | 3.3 |
| 1994 | –1.9 | 4.9 | 0.2 | 3.4 |
| 1995 | –1.9 | 5.2 | 0.0 | 3.6 |
| 1996 | –2.1 | 5.0 | 0.0 | 3.6 |
| Total | **–1.9** | **4.9** | **0.2** | **3.4** |

### *Extreme model performance*

We will no longer consider the Altshuler and UNB1 models in this paper because their performance with regard to large, or extreme, residuals is poor.

Comparison of Figure 1 and Figure 2 reveals the 40% improvement the UNB3 models provide over the Altshuler model at the upper 4σ level.

It is convenient to use a "non-extreme" cut-off range of ±20 cm for the UNB3 models, based on the results shown in Figure 2. A zenith delay error of this magnitude could lead to a potential 2 m bias in height (see later discussion). The numbers of residuals per year exceeding this range from the UNB3 models are given in Table 1. In total, there are 72 residuals from the UNB3 model exceeding the ±20 cm range out of 1,011,413 profiles. This is equivalent to approximately 0.007 %. Correspondingly, 99.99288 % of the residuals are within the non-extreme range.

Table 2. Number of residuals exceeding ±20 cm for UNB3 and UNB3(SfcMet) models.

| Year | UNB3 | | UNB3(SfcMet) | |
|---|---|---|---|---|
| | –Max | +Max | –Max | +Max |
| 1987 | 0 | 3 | 0 | 5 |
| 1988 | 0 | 7 | 2 | 15 |
| 1989 | 0 | 1 | 2 | 6 |
| 1990 | 0 | 3 | 0 | 5 |
| 1991 | 2 | 0 | 3 | 2 |
| 1992 | 9 | 2 | 0 | 3 |
| 1993 | 9 | 1 | 4 | 5 |
| 1994 | 8 | 2 | 6 | 3 |
| 1995 | 9 | 8 | 11 | 10 |
| 1996 | 4 | 4 | 14 | 10 |
| Total | **72** | | **106** | |

Table 2 also shows that the UNB3(SfcMet) model is more susceptible to extremes than the UNB3 model. In general, the same extremes show up for both models, however the UNB3(SfcMet) model is sensitive to measured values of surface temperature and water vapour that, when combined with the empirical lambda parameter, are unrepresentative of the water vapour profile as a whole. As Table 1 shows, the mean error of the UNB3(SfcMet) model may be near-zero, but its overall distribution, according to Figure 2, is slightly less Normal than the UNB3 model.

Some of the extremes recorded with UNB3(SfcMet) may be due to incorrect surface measurements. All the initially detected "extreme" profiles have been checked and several were rejected as unlikely. Unfortunately, without detailed knowledge of the climatic conditions at each station, it is not possible to be completely sure about all the remaining extremes. However, poor instrument quality is a potentially important factor when using real-time data and the impact cannot always be quantified and any resulting errors rectified. Hence, is felt that the remaining extremes are representative of using real-time meteorological data.

Because the focus of our work is on the WAAS environment and because wide-area differential aircraft receivers generally do not have access to real-time measurements of the atmosphere, we will concentrate solely on the UNB3 model using the look-up table. As Table 2 shows, there is no obvious gain to be made, with regard to extreme errors, in replacing a good meteorological look-up table with real-time values. The small improvement in the overall bias is negligible when compared to other potential error sources, such as multipath and the ionosphere.

The location of stations with extreme residuals is shown in Figure 3. Stations with at least one positive extreme (a residual greater than +20 cm zenith delay error) are shown as a triangle and labeled with a station identification number to the right. Stations with at least one negative extreme (a residual magnitude greater than –20 cm zenith delay error) are shown as an inverted triangle and labeled to the left. Parenthetical numbers indicate more than one extreme over the ten year period.

Figure 3 indicates that the negative extremes are confined to the Baja California, Sonora and Sinaloa regions of Mexico and the southern tips of California and Arizona. The number and location of the positive extremes are geographically more scattered than the negative extremes, although concentrations do occur. Bermuda (station number 13601), for example, seems particularly prone to extreme conditions, a possible consequence of its mid-Atlantic location.

It is useful to examine a plot of the extremes versus day-of-year, especially to understand the pattern of negative extremes. Figure 4 shows that most of the negative extremes occur during the late spring. Examining the residual time series for stations in the west of Mexico indicates that the climate continues to be relatively constant through this period. Unfortunately, the sinusoidal variation in the UNB3 model is increasing at this time and hence the error for these stations can exceed the ±20 cm error limit. The positive extremes are more confined to the summer season, although the second and third largest extremes occurred outside of this period, in the winter. Unlike the majority of negative extremes, the positive extremes are outliers in the overall time series, suggesting the influence of short period, transient weather systems.

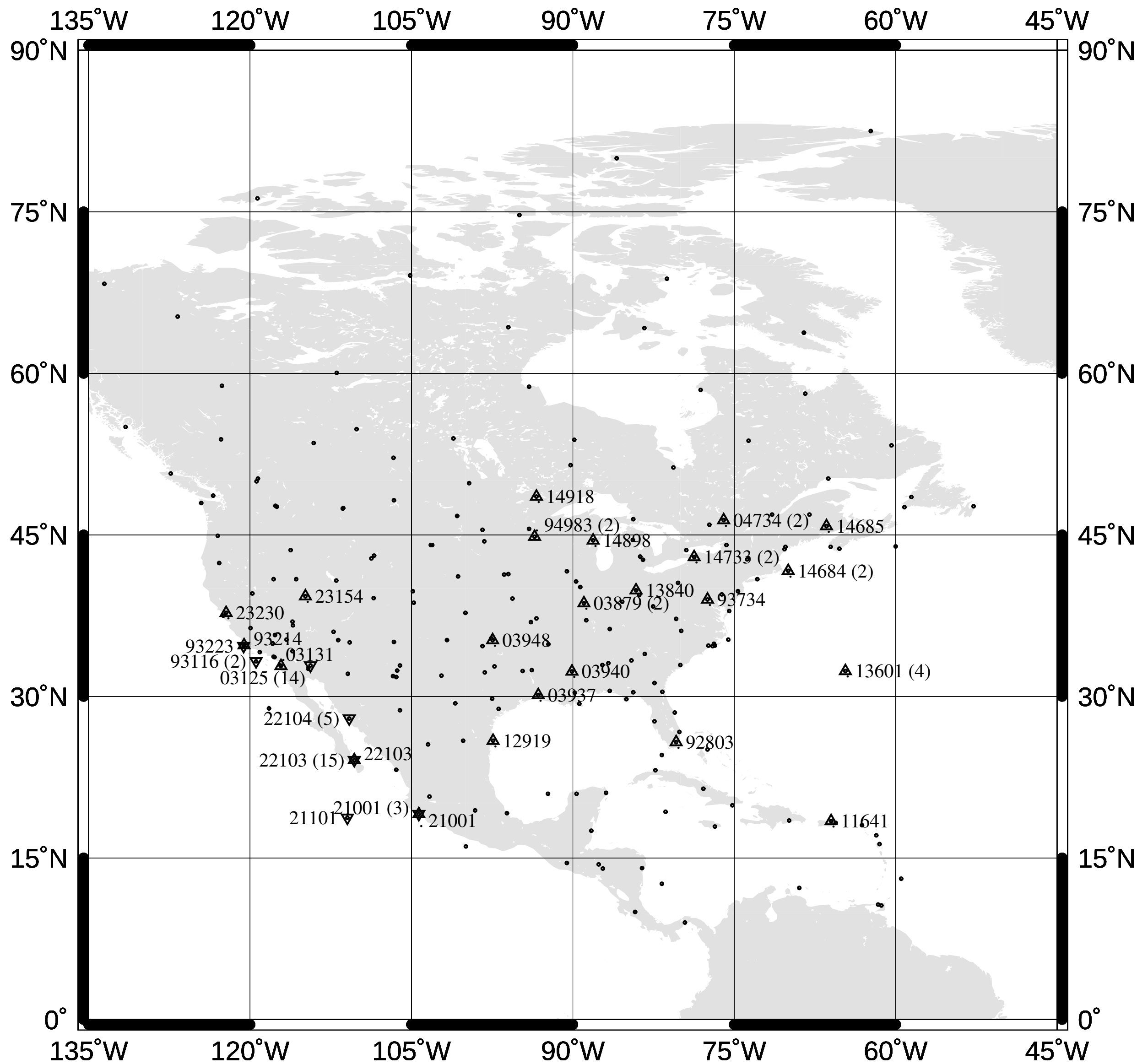


Figure 3. Location of radiosonde stations providing data between 1987 and 1996. Stations with positive extremes from model UNB3 are labeled ▲ and stations with negative extremes are labeled ▼ along with their identification numbers. Parenthetical digits indicate number of extremes for sites with more than one.

To examine the maximum extreme residuals more closely, we have plotted the first and last ten ordered residuals (in terms of magnitude) for each year in Figure 5. This plot shows that the largest positive extreme varies from year to year, and need not necessarily exceed +20 cm, as in 1991 for example. This indicates the danger of using only one year's-worth of data to study extremes. The number of extremes per year also varies. The largest extreme in our data set (42 cm) occurred in 1995 at station 22103 (La Paz, Mexico) during the passage of a tropical cyclone (Hurricane Flossie) and represents a wet zenith delay of over 70 cm, which is extremely large. We are attempting to verify the validity of both this, and the other extremes, with an independent data source.

The negative extremes follow a very intriguing pattern, one that changes approximately with the total number of stations operating each year. Figure 3 reveals that stations 03125 (Yuma, Ariz.) and 22103 (La Paz) contribute almost two-thirds of these residuals and it should be pointed out that neither station existed before 1992 and 1991 respectively. However some kind of climatic influence cannot be ruled out and requires further investigation.

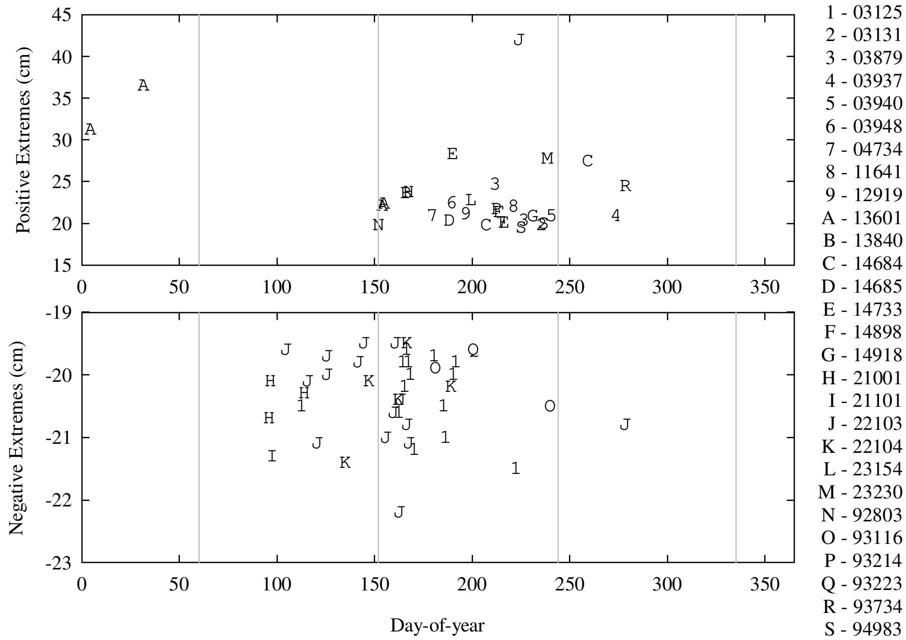


Figure 4. Time of extremes during yearly period. Approximate extent of the seasons is delineated with vertical lines. Key to the right links plotted character codes with station identification codes used in Figure 3.

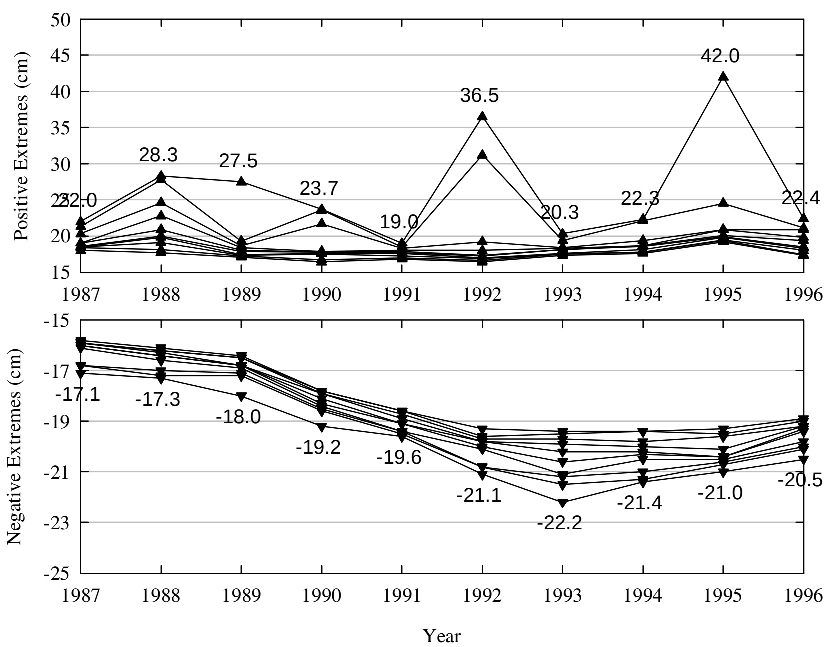


Figure 5. Ten largest positive and negative UNB3 zenith delay errors each year. The magnitude of the maximum residual in each year is labeled (in cm).

***Extreme value prediction***

Given that Figure 2 indicates different tail distributions from that of the Normal distribution, we can examine the extreme residuals separately using extreme value distributions (Gumbel, Frechet and Weibull distributions). These distributions are the limiting forms that most common distributions take when only considering the largest or smallest values in a set of samples. It is not necessary to know the underlying distribution to consider the distribution of the extreme values [*Castillo*, 1988].

An extreme value probability plot has the potential to provide a lot of useful information. Figure 6 and Figure 7 show the extreme value probability plots for both the largest yearly positive and negative extremes respectively. Of the three distributions, the positive extremes fit a Frechet distribution best and the negative extremes fit a Weibull distribution best.

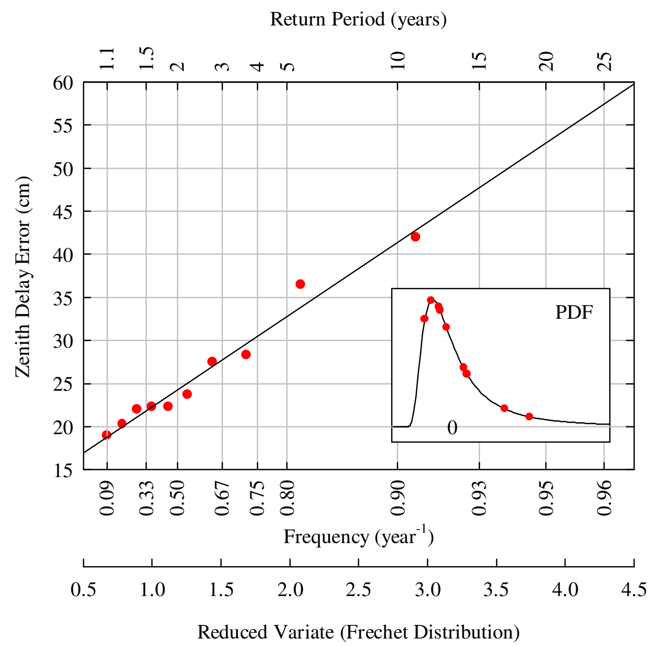


Figure 6. Extreme value cumulative probability plot for positive extremes.

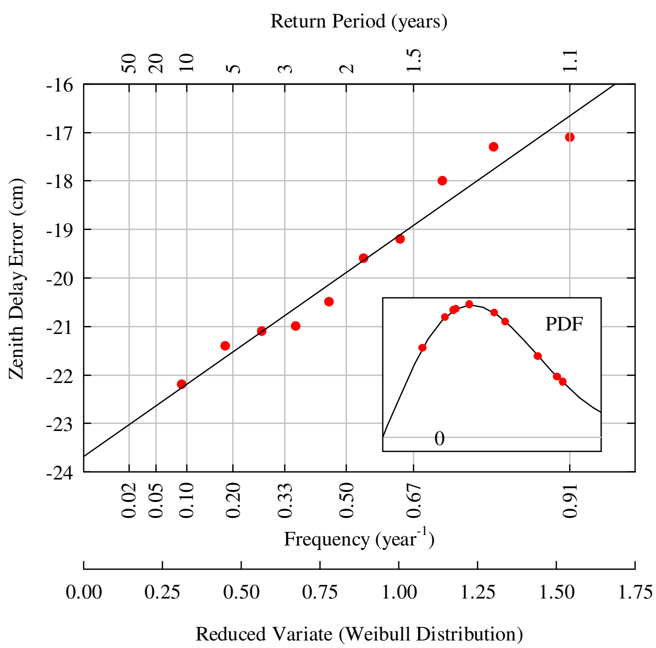


Figure 7. Extreme value cumulative probability plot for negative extremes.

The power in these plots lies in the ability to extrapolate and compute the return periods of future extremes. The return period is merely the inverse of the expected frequency of occurrence. By using the largest value in each year, the return periods are given in units of years. It should be pointed out that the use of extreme value statistics usually requires a set of at least 20 samples from which to draw the extreme values, and that with only ten years of data in our sample, the confidence in these results is not high. However, we are able to give a good example of the application of extreme value statistics.

If the positive extremes do follow the Frechet distribution, then an average return period of 25 years is forecast for an extreme zenith delay value of 57.5 cm. Both the Frechet and Weibull distributions are specified with threshold values beyond which the expected frequency is one or zero depending on whether positive or negative extremes are being investigated. The Frechet threshold for the positive extremes is approximately 11 cm, which indicates that an error at least this large will occur every year.

If we assume no error in the hydrostatic delay, the negative extremes are limited in magnitude by the maximum wet zenith delay value of the UNB3 model (~27 cm). This error would occur with a dry, or nearly dry, atmosphere in the tropics. Hence, we could specify this value as the threshold in the Weibull distribution, however we have tried to use the data itself to identify the value. This appears to work because the Weibull distribution best fit specifies a cut-off value of –23.7 cm. This slight difference suggests either insufficient data, or that the maximum zenith wet delay error will never be reached because there is always some water vapour (i.e. at least a few centimetres worth of wet zenith delay) in the atmosphere, which is almost certainly true. The forecast return period for an extreme zenith delay error less (i.e. greater in absolute value) than –23 cm is 50 years.

***Impact on position determination***

The impact of an unmodelled tropospheric range delay on the GPS position determination is complicated by the elevation angle dependence of the error. The value will not be constant for all the satellites in view and hence the VDOP cannot be reliably used. The only way to study the impact is to undertake position simulations, replacing the GPS range with the unmodelled delay. In this way we can predict how the error is mapped into the position coordinates.

We have computed position solution biases for all the stations with extreme residuals. Broadcast ephemerides were used from 1997 to provide satellite constellations for six hours around the time of the radiosonde launches.

In theory, the extreme residuals we have seen could occur at any time of year at the stations, however to perform position simulations for each day of the year would be time consuming. In addition, we have assumed that the tropospheric error remains constant over short periods of several hours. In any case, it is possible to derive a general relationship between tropospheric delay error and the resulting position bias, as we shall see.

Two kinds of position solution simulations were performed — a regular unweighted least-squares solution and a weighted least-squares solution using the inverse of the square of the mapping function to down-weight any low elevation angle errors. The position biases were computed every two minutes. For almost all of the position solutions, the weighted vertical biases were from one-third to two-thirds smaller than the unweighted vertical biases. In general the weighted solution reduced the extreme vertical bias to the metre level or less. The horizontal biases for both solutions were always much smaller — at the decimetre level or less.

It was discovered that for one particular time period at one station, the satellite constellation was dominated by low elevation angle satellites. Over this period (approximately 10 minutes), the weighted and unweighted position solutions converge towards the same values (~1.5 m). This is can be seen in Figure 8, as can the correlation between the unweighted vertical position bias and the maximum tropospheric delay error. It is also clear that any function of VDOP will not correctly model this kind of bias. The residual zenith error for this station was approximately 21 cm and there is one time period where the unweighted vertical bias approaches 2 m.

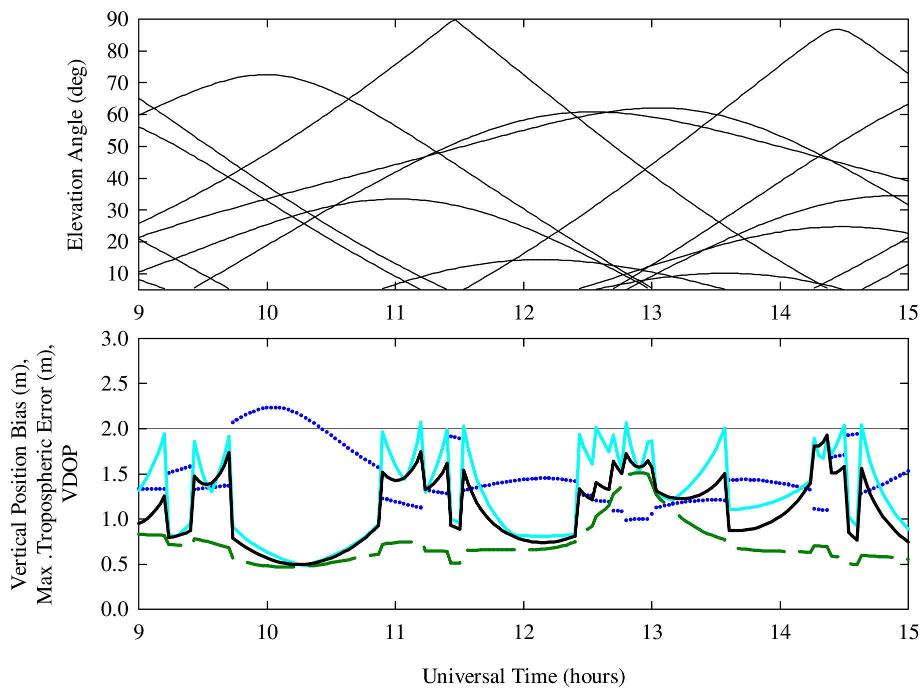


Figure 8. Satellite constellation at station 12919, Brownsville, Texas, July 18, 1997 and simulated vertical position biases from a zenith delay error of 21 cm. Solid line — unweighted solution bias; dashed line — weighted solution bias; light line — maximum tropospheric error (error at lowest elevation satellite); dotted line — VDOP.

This result points to a general rule to indicate the amount of vertical position bias due to an unmodelled tropospheric range delay. This will be approximately the same value as the maximum residual tropospheric delay present in the solution. Due to the elevation angle dependence of the tropospheric delay, this will essentially be the delay error on the lowest-elevation angle satellite. Given the expected zenith delay error, an approximate mapping function value will give the correct result. For an overall "rule-of-thumb", the mapping function value at 5 degrees elevation angle can be taken to be 10, and the maximum value of the possible vertical height error can be easily calculated.

The size of the maximum bias in the computed position, will be limited in one direction because of the lower limit of the UNB3 model. A negative error indicates that the tropospheric delay model prediction was too large. By effectively shortening the range, the computed position will be higher than the true position. Hence an aircraft flying below its intended height can only ever be approximately 3 m too low at most due to tropospheric delay mis-modelling. For an aircraft flying above its intended height, given an unfavourable satellite constellation and unusual weather conditions, vertical position biases of upto 4 metres are possible for wide-area differential users, due solely to mis-modelled tropospheric delays. Errors of 5 metres are predicted by the extreme value theory.

## CONCLUSIONS

Our results indicate that the maximum size of the residual tropospheric delay is not too large, as long as a good model is used. For the UNB3 model, only 7 in 100,000 predictions resulted in residual zenith delay errors outside the range of ±20 cm. The distribution of the majority of the errors can be adequately represented by a zero-mean Normal distribution with standard deviation of 5 cm upto the ±4σ point. Beyond this, the Normal distribution is too conservative for the positive extreme residuals. Of the extreme values beyond ±20 cm, there are slightly more negative extremes than positive. The negative extremes appear more consistent with prevailing climatic conditions and are limited in magnitude by the maximum wet zenith delay value of the UNB3 model (~27 cm). There is more potential for greater positive extremes, but a reliable forecast is limited by the small sample (10 values). More data (i.e. from additional years) would be required to improve the confidence of future predictions.

The impact of a mis-modelled tropospheric range delay on the computed position is primarily confined to the height component. The VDOP value is not a good indicator of the vertical bias because it effectively down-weights the contribution of low elevation satellites where the error is greatest. A better indication is provided by the maximum residual error, i.e. the unmodelled range on the lowest elevation satellite. This error maps almost directly into the vertical position component.

## ACKNOWLEDGMENTS

This work has been undertaken on behalf of Nav Canada in conjunction with the U.S. Federal Aviation Administration.

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