Minding Your *I*s and *Q*s

A radio signal may be represented, in general, as

$$x(t) = A\cos(\omega t + \varphi)$$

where A is the instantaneous amplitude, ω is the instantaneous carrier frequency in radians per second and φ is the instantaneous phase offset. Depending on the modulation type, the amplitude, frequency, and or/phase can be functions of time.

How can we determine the amplitude and phase of an unknown signal if we only know its frequency? A common approach starts by multiplying the signal by a pair of reference signals

$$\cos(\omega t) \text{ and } \cos(\omega t + 90°) = -\sin(\omega t)\,.$$

These reference signals are 90° out of phase with each other. In other words, they are orthogonal or in phase quadrature.

The resulting signals are

$$\begin{aligned} I &= A\cos(\omega t + \varphi)\cos(\omega t) \\ Q &= -A\cos(\omega t + \varphi)\sin(\omega t) \end{aligned}$$

Using a bit of trigonometry, we find that

$$\begin{aligned} I &= \frac{1}{2}A\left[\cos(\omega t + \varphi - \omega t) + \cos(\omega t + \varphi + \omega t)\right] \\ &= \frac{1}{2}A\left[\cos(\varphi) + \cos(2\omega t + \varphi)\right] \\ Q &= \frac{1}{2}A\left[\sin(\omega t + \varphi - \omega t) - \sin(\omega t + \varphi + \omega t)\right] \\ &= \frac{1}{2}A\left[\sin(\varphi) - \sin(2\omega t + \varphi)\right] \end{aligned}$$

with the signal products consisting of a zero frequency (direct current or DC) component and one with a frequency equal to twice the signal frequency. If these signals are low-pass filtered, we obtain

$$\begin{aligned} I &= \frac{1}{2}A\cos\varphi \\ Q &= \frac{1}{2}A\sin\varphi \end{aligned}\,.$$

I and Q are known as the in-phase and quadrature-phase (or quadra-phase) signal components respectively. This is because I is maximized when the unknown signal is phase aligned with cos(ωt) and Q is maximized when the unknown signal is phase aligned with –sin(ωt).

The amplitude of the unknown signal can then be determined using the vector magnitude of the I and Q components:

$$A = 2\sqrt{I^2 + Q^2}$$

while the phase can be determined using the four-quadrant arctangent of the ratio of the Q and I components:

$$\varphi = \tan^{-1}\left(\frac{Q}{I}\right).$$

(Other sign conventions for the reference signals and I and Q components are possible with the same results.)

If a radio receiver provides both in-phase and quadrature-phase local oscillator signals, then an amplitude modulated (AM) radio signal can be demodulated using the vector magnitude of the filtered I and Q intermediate-frequency products. In fact, signals using any of the common modulation techniques can be demodulated by manipulating the I and Q products. The approach works whether using analog or digitized signals.

In a digital GPS receiver, after suitable down conversion of the received signal and digitization, in-phase and quadrature-phase sampled data are produced using I and Q replica carriers (including carrier Doppler shift) from a numerical (digitally) controlled oscillator. The I and Q signals are correlated separately with early, prompt, and late replica pseudorandom noise codes (plus code Doppler) synthesized by the code generator. Just two (prompt and "dithered") replicas can be used if the dithered replica can be selected to be early or late. After integration of the resulting signals, baseband processing takes place using code and carrier tracking loop discriminators and loop filters to provide correction signals for the code and phase tracking loops and to provide measurements of the replica code phase and replica carrier Doppler phase which can be converted into pseudorange and carrier-phase measurements for positioning, navigation, and timing solutions.

A transmitted radio signal itself may be include separate I and Q components. For example, the GPS L1 signal is modulated with the C/A-code and the navigation message on the in-phase channel and the P(Y)-code and the navigation message on the quadrature channel. – R.B.L.