THE EFFECT OF THE IONOSPHERE AND TROPOSPHERE ON SATELLITE POSITIONING SYSTEMS*

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ABSTRACT. The range and range-rate measurements obtained from satellite positioning systems are biased by the refraction of the earth's atmosphere. For radiometric measurements, the effect is twofold. Radio waves emitted by satellites must first traverse the ionosphere where the refractive index differs from unity by an amount that is, to a very good approximation, proportional to electron density and inversely proportional to the square of the carrier wave frequency. At the 1575.42 MHz frequency of the L1 signal emitted by the satellites of the Navstar Global Positioning System (GPS), for example, the measured range of a vertically propagating signal can be biased by the ionosphere by up to 30 metres or so. The non-ionized part of the earth's atmosphere also takes its toll on radio signals. The refractive index of the non-ionized atmosphere (primarily the troposphere and to a much lesser extent also the stratosphere) is governed by the density of the dry gases and water molecules therein. Below about 30 GHz, the non-ionized atmosphere is non-dispersive (except for the anomalous dispersion of the water vapour and oxygen spectral lines) and so its effect on radio waves is independent of their frequency. Vertically propagating radio signals experience a propagation delay of roughly 2.3 metres. Systems using visible light waves, such as satellite laser ranging, are immune from ionospheric effects but have to contend with a dispersive troposphere. The density of electrons, atoms, and molecules in the atmosphere is both spatially and temporally variable. This variation makes the modelling of biases experienced by transatmospheric electromagnetic signals a challenging exercise. In this paper, I review the attempts being made to understand and model the effects of the atmosphere on the signals of satellite positioning systems including GPS, GLONASS, the Navy Navigation Satellite System (also know as Navsat and Transit), Tsikada, Argos, COSPAS-SARSAT, DORIS, PRARE, and satellite laser ranging.

1. INTRODUCTION

While the earth's atmosphere is indispensable for the existence of life on the planet, it is less than a benefit for the users of satellite-based positioning systems. Electromagnetic signals propagating from a satellite to the ground or vice-versa transit both the ionosphere and troposphere and both of these media exact a toll on the signals in the way of a bias on measured ranges and range rates. In order to interpret the measurements as true geometric quantities, one must account for these biases in the processing of the data.

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Ranges may be inferred by measuring the time delay in the propagation between the satellite and the ground of a signal modulation. Such measurements are biased by timing errors as well as atmospheric and other effects and so are usually referred to as pseudoranges. Ranges can also be determined from measured phases of a pure carrier wave. Such measurements are generally more precise than pseudoranges but the inherent integer cycle ambiguity associated with such measurements must be resolved. The Global Positioning System (GPS) pseudorange and carrier phase observation equations, for example, are

$$p = \rho + c(dt_s - dt_r) + d_{ion} + d_{trop} + dp_m + \varepsilon_p \quad , \tag{1}$$

$$\Phi = \rho + c(dt_s - dt_r) + \lambda N - d_{ion} + d_{trop} + d\Phi_m + \varepsilon_\Phi \quad , \tag{2}$$

where

where	p	is the pseudorange (m)
	ρ	is the geometric range from the receiver to the satellite (m)
	c	is the speed of light in vacuum = 299 792 458 m/sec
	dt_s	is the satellite clock offset (seconds)
	dt_r	is the receiver clock offset (seconds)
	d_{ion}	is the ionospheric delay (m)
	d_{trop}	is the tropospheric delay (m)
	dp_m	is the effect of multipath on the pseudorange (m)
	ε_p	is the pseudorange random error (m)
	Φ	is the continuous carrier phase (m)
	λ	is the signal wavelength (m)
	N	is the carrier phase ambiguity (cycles)
	$d\Phi_m$	is the effect of multipath on the continuous carrier phase (m)
	ε_Φ	is the continuous carrier phase random error (m).

Of concern to us in this paper are the d_{ion} and d_{trop} bias terms.

A third measurement type used in some satellite positioning systems is the time rate of change of carrier phase which is equivalent to the Doppler shift of the carrier.

In this paper, I will briefly describe the effect of the ionosphere and troposphere on satellite positioning systems and summarize some of the recent progress that has been made in the attempt to improve the accuracy of models of the biases. I will restrict my attention to a discussion of the effects on pseudorange and phase measurements of microwave systems, principally the Navstar Global Positioning System but the discussions will hold equally well for the former Soviet Union's Global Navigation Satellite System, GLONASS, which utilizes similar signals [Kleusberg, 1990] and a number of other proposed global navigation satellite systems. The discussion is also relevant for the Precise Range and Range-Rate Equipment (PRARE) system [Flechtner, 1990] although this system provides two-way (ground to satellite to ground) as well as a one-way (satellite to ground) measurements.

Several satellite positioning systems have been developed which measure single or dual frequency Doppler shifts of carrier frequencies. The grandfather of these systems is the dual-frequency United States Navy Navigation Satellite System (also know as Navsat and Transit) [Pryor, 1989]. The former Soviet Union's Tsikada system [Wood and Perry, 1980; anon., 1986] operates on a similar principle. The single frequency Argos [Service Argos, 1978] and COSPAS-SARSAT [McPherson, 1981] systems measure Doppler shifts of signals transmitted from ground stations as they are received at a satellite. This technique, in dual-frequency form, has been refined in the Doppler Orbitography and Radiopositioning Integrated on Satellite (DORIS) system [Willis et al., 1990]. The effect of the ionosphere and troposphere on the measurements of Doppler shift can be assessed simply by taking the time derivative of the carrier phase effects.

Also not specifically mentioned directly in this paper is the effect of the ionosphere and troposphere on satellite microwave radar altimetry systems. But again, the effect on these systems can be inferred from the ensuing discussion of the effects on GPS.

I will conclude the paper with a brief discussion of the effect of the troposphere on satellite laser ranging (SLR) measurements. The refractive index of air at visible wavelengths, such as those used by SLR, and at radio wavelengths can be significantly different. The reasons for this difference will be briefly described.

The effect of the ionosphere and troposphere on radiometric satellite positioning systems is shared by very long baseline interferometry (VLBI). Much of the research taking place to understand and model the effect of the atmosphere on VLBI is applicable to GPS and I shall refer to some of this research in the discussion below.

Several comprehensive reviews on the effect of the atmosphere on satellite positioning systems have been published recently. The monograph on atmospheric effects on geodetic space measurements edited by Brunner [1988] describes significant advances in the understanding and modelling of refraction effects on space measurements. Brunner [1991] has also recently compiled a report on the progress in the study of wave propagation in refractive media since 1988. Dixon [1991] includes a discussion of the effects of the ionosphere and troposphere in his comprehensive review paper on the use of GPS for geodynamical studies.

2. THE IONOSPHERE

The ionosphere is that region of the earth's atmosphere in which ionizing radiation (principally from solar ultraviolet and x-ray emissions) causes electrons to exist in sufficient quantities to affect the propagation of radio waves. This definition does not impose specific limits on the height of the ionosphere. Nevertheless, it is useful to delineate some sort of boundary to the region. The height at which the ionosphere starts to become sensible is about 50 km and it stretches to heights of 1,000 km or more. Indeed, some would argue for an upper limit of 2,000 km. The upper boundary depends on what particular plasma density one uses in the definition since the ionosphere can be interpreted as thinning into the interplanetary plasma. Although the interplanetary plasma affects the propagation of the signals from space probes and the quasar signals observed in very long baseline radio interferometry, it may be considered to lie beyond the orbits of satellites used in positioning systems and therefore will be ignored in this paper.

The ionosphere is a dispersive medium for radio waves; that is, its refractive index is a function of the frequency. The refractive index is given by the Appleton-Hartree theory of an ionised medium in which there are an equal number of positive ions and free electrons. It is assumed that a uniform magnetic field is present and that the ions have negligible effect on radio waves. The complex refractive index, n, at angular frequency, ω, is given by [e.g., Bradley, 1989]

$$n^2 = (\mu - i\chi)^2$$

$$= 1 - \frac{X}{1 - iZ - \frac{Y_T^2}{2(1 - X - iZ)} \pm \left(\frac{Y_T^4}{4(1 - X - iZ)^2} + Y_L^2 \right)^{\frac{1}{2}}} \tag{3}$$

where

$$X = \frac{Ne^2}{\varepsilon_0 m\omega^2} \quad Y_L = \frac{eB_L}{m\omega} \quad Y_T = \frac{eB_T}{m\omega} \quad Z = \frac{\nu}{\omega}.$$

In equation (3), N is the electron density, e and m are the electron charge and mass respectively, ε_0 is the permittivity of free space, and n is the electron collision frequency. The subscripts T and L refer to the transverse and longitudinal components of the earth's magnetic field B. The quantity

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} \frac{Ne^2}{\varepsilon_0 m} \tag{4}$$

is known as the electron plasma resonance frequency. Typically, $f_0 < 30$ MHz. For frequencies $f >> f_0$, such as those used by GPS, typical values for the components of the earth's magnetic field, and ignoring electron collisions, the refractive index can be well approximated by

$$n = 1 - \frac{X}{2(1 \pm Y_L)}. \tag{5}$$

The refractive index may be further approximated by ignoring the effect of the longitudinal components of the earth's magnetic field. If this is done, to first order, the phase refractive index of the ionosphere, appropriate for carrier phase observations, is given by

$$n_\phi = 1 - \alpha N / f^2 \tag{6}$$

where α is a constant. Since group refractive index is defined as

$$n_g = n + f \frac{dn}{df} \tag{7}$$

we have for the ionospheric group refractive index, appropriate for pseudorange observations,

$$n_p = 1 + \alpha N / f^2. \tag{8}$$

If N has units of reciprocal metres cubed and f is given in Hz, then α has the value 40.28.

The integration of these expressions along the path followed by a radio signal yields the electromagnetic path lengths

$$\rho_\phi \;=\; \int (1 - \alpha\, N \,/\, f^2)\; ds \;=\; \rho \;-\; d_{ion} \tag{9}$$

and

$$\rho_p \;=\; \int (1 + \alpha\, N \,/\, f^2)\; ds \;=\; \rho \;+\; d_{ion} \tag{10}$$

where ρ is the true geometric range and d_{ion} is the ionospheric range error which is given by

$$d_{ion} \;=\; \alpha\, TEC \,/\, f^2 \tag{11}$$

with TEC (Total Electron Content) being the integrated electron density along the signal path. Carrier phase measurements of the range between a satellite and the ground are reduced by the presence of the ionosphere (the phase is advanced) whereas pseudorange measurements are increased (the signal is delayed) — by the same amount. TEC is highly variable both temporally and spatially. The dominant variability is diurnal. There are also solar-cycle and seasonal periodicities as well as short-term variations with common periods of 20 to over 100 minutes. Typical daytime values of TEC for mid-latitude sites are of the order of 10^{18} m^{-2} with corresponding night-time values of the order of 10^{17} m^{-2}. However, such typical day-time values can be exceeded by a factor of two or more, especially in near-equatorial regions. For a discussion of the variability of TEC values, see Jursa [1985].

Values for d_{ion} at the GPS L1 frequency of 1575.42 MHz in the zenith direction can reach 30 metres or more and near the horizon this effect is amplified by a factor of about three.

Dual frequency positioning systems take advantage of the dispersive nature of the ionosphere for correcting for its effect. In the case of GPS, for example, a linear combination of the L1 and L2 pseudorange measurements may be formed to estimate and subsequently remove the ionospheric bias from the measurements:

$$d_{ion}(L1) \;=\; \frac{f_2^2}{f_2^2 - f_1^2}\;[\, p_1 - p_2 \,] \;+\; \varepsilon_{p12} \tag{12}$$

where f_1 and f_2 are the L1 and L2 carrier frequencies respectively, p_1 and p_2 are the L1 and L2 pseudorange measurements, and ε_{p12} represents random measurement errors and unmodelled biases. A similar approach is used to correct carrier phase measurements with

$$d_{ion}(L1) \;=\; \frac{f_2^2}{f_2^2 - f_1^2}\;[\, (l_1\, N_1 - l_2\, N_2) - (F_1 - F_2) \,] \;+\; \varepsilon_{F12} \tag{13}$$

where F_1 and F_2 are the L1 and L2 carrier phase measurements (in units of length) respectively, l_1 and l_2 are the L1 and L2 carrier wavelengths respectively, N_1 and N_2 are the L1 and L2 integer cycle ambiguities respectively, and ε_{F12} represents random measurement errors and unmodelled biases. In practice, N_1 and N_2 cannot be determined but as long as the phase measurements are continuous (no cycle slips) they remain constant. Hence the carrier phase measurements can be used to determine the variation in the ionospheric delay — the so-called differential delay — but not the absolute delay at any one epoch. The estimation of the differential delay in this way (having ignored third and higher order effects) is good to a few centimetres. Brunner and Gu [1991] have proposed an improved model which accounts for the higher order terms neglected in the first order approximation, the geomagnetic field effect, and ray path bending. Numerical simulations showed that the residual range errors associated with the new model are less than two millimetres.

Note that in the dual-frequency correction approach, it is assumed that the L1 and L2 signals follow the same path through the ionosphere. While this is not quite true (at an elevation angle of 15°, for example, the maximum separation of the raypaths for a high TEC value of 1.38×10^{18} m^{-2} is about 35 metres [Brunner and Gu, 1991]), the error induced is generally negligible except under conditions of severe ionospheric turbulence.

If measurements are made at only one carrier frequency, then an alternative procedure for correcting for ionospheric bias must be used. The simplest approach, of course, is to ignore the effect. This is what is typically done by navigators, for example, using single frequency Transit receivers. This approach is also followed by surveyors carrying out relative positioning using single frequency GPS receivers. Differencing between the observations made by simultaneously observing receivers removes that part of the ionospheric range error that is common to the measurements at both stations. The remaining residual ionospheric range error results from the fact that the signals received at the two stations have passed through the ionosphere at slightly different zenith angles. Therefore, the TEC along the two signal paths is slightly different, even if the vertical ionospheric profile is identical at the two stations. It has been shown [e.g. Georgiadou and Kleusberg, 1988] that the main result of this effect in differential positioning is a baseline shortening proportional to the TEC and proportional to the baseline length. Beutler et al. [1988] and Santerre [1989, 1991] have examined the effect of the GPS satellite sky distribution on the propagation of residual ionospheric errors. Such errors can introduce significant scale and orientation biases in relative coordinates. For example, at a typical mid-latitude site using an elevation cut-off angle of 20°, a horizontal scale bias of -0.63 parts per million is incurred for each 1×10^{17} m^{-2} of TEC not accounted for.

It is also possible to use an empirical model to correct for ionospheric bias. The GPS navigation signal, for example, includes the parameters of a simple prediction model [Klobuchar,1986]. Recent tests of this model against a limited set of dual frequency GPS data showed that this broadcast model can perform very well. Newby and Langley [1992] showed that the model accounted for approximately 70 to 90% of the daytime ionospheric delay and 60 to 70% of the night-time delay at a mid-latitude site during a time of high solar activity. These results indicate that the broadcast model can, at times, remove more than the 50 to 60% r.m.s. of the ionosphere's effect generally acknowledged as the performance level of the model [e.g., Klobuchar, 1986; Feess and Stephens, 1986]. This same study showed that more sophisticated ionospheric models (the Bent Ionospheric Model [Bent and Llewellyn, 1973], the 1986 International Reference Ionosphere (IRI) [Rawer et al., 1981], and the Ionospheric Conductivity and Electron Density Profile (ICED) [Tascoine et al., 1988] — see also Bilitza [1990]) did not appear to perform significantly better, on average, than the broadcast model. In fact, the performance of the ICED model was markedly poorer. Brown et al. [1991] have also evaluated the usefulness of ionospheric models as predictors of TEC. They concluded that none of the six models tested do a very good job probably because the top part of the ionosphere is inaccurately represented. Leitinger and Putz [1988] have looked at the use of the Bent and IRI models in providing information for higher order corrections of the ionospheric range bias.

Georgiadou and Kleusberg [1988] developed a model for the correction of carrier phase GPS observations from a network of single frequency receivers using estimated vertical ionospheric biases derived from the observations of a dual frequency receiver in the vicinity of the network. Webster and Kleusberg [1992] have recently extended this technique to correct the observations from an airborne single frequency receiver moving in the vicinity of three ground-based dual frequency receivers. A similar approach has been followed by Wild et al. [1989].

2.1 Scintillation and Magnetic Storms

If the number of electrons along a signal path from a satellite to a receiver changes rapidly, the resulting rapid change in the phase of the carrier may present difficulties for the carrier tracking loop in the receiver. For a GPS receiver tracking the L1 signal, a change of only 1 radian of phase (corresponding to 0.19 x 10^{16} el/m^2 change in TEC, or only 0.2% of a typical 10^{18} el/m^2 TEC) in a time interval equal to the inverse of the receiver bandwidth is enough to cause problems for the receiver's tracking loop. If the receiver bandwidth is only 1 Hz (which is just wide enough to accommodate the geometric Doppler shift) then when the second derivative of the phase exceeds 1 Hz per second, loss of lock will result. During such occurrences, the amplitude of the signal is generally fading also. These short-term (1 to 15 seconds) variations in the amplitude and phase of signals are known as ionospheric scintillations.

The loss of lock results in a phase discontinuity or cycle slip. A cycle slip must be repaired before the data following the slip can be used. Large variations in ionospheric range bias over short intervals of time can make the determination of the correct integer number of cycles associated with these phase discontinuities difficult. If the variations of the ionospheric range bias exceed one half of a carrier cycle, they may be wrongly interpreted in the data processing as a cycle slip.

There are two regions where irregularities in the earth's ionosphere often occur causing short term signal fading which can severely test the tracking capabilities of a GPS receiver: the region extending ±30° either side of the geomagnetic equator and the auroral and polar cap regions [see, e.g., Héroux and Kleusberg, 1989]. The fading can be so severe that the signal level drops completely below the signal lock threshold of the receiver. When this occurs, data is lost until the receiver reacquires the signal. The process of loss and re-acquisition of signals may go on for several hours.

Such signal fading is also associated with geomagnetic storms. Magnetic storms (and the associated ionospheric storms) occur when high-energy charged particles from solar flares, eruptive prominences, or coronal holes arrive at the earth causing perturbations in the earth's magnetic field. The charged particles interact with the earth's neutral atmosphere producing excited ions and additional electrons. The strong electric fields that are generated cause significant changes to the morphology of the ionosphere, greatly changing the propagation delay of GPS pseudoranges and the advance in the carrier phases within time intervals as short as one minute. Such changes in the polar and auroral ionospheres can last for several hours.

Occasionally magnetic storm effects extend to the mid-latitudes. During the magnetic storm that occurred in March 1989, range-rate changes produced by rapid variations in TEC exceeded 1 Hz in one second [Klobuchar, 1991]. As a result, GPS receivers with a narrow 1 Hz bandwidth were continuously losing lock during the worst part of the storm because of their inability to follow the changes.

2.1.1 Testing of GPS Receiver Performance During Hightened Ionospheric Activity

Few dedicated tests of the behaviour of GPS receivers during periods of scintillation or magnetic storms have been carried out although there are many reports in the literature of satellite tracking problems during survey campaigns which were believed to be due to high ionospheric activity. Rocken et al. [1990] report on a tracking test of Trimble 4000 SST and Texas Instruments TI 4100 receivers carried out on Ascension Island which lies in the equatorial region. The reason for this particular test was to see if the new (at the time) 4000 SST performed better than the 4000 SD which had been used during a GPS campaign in the south Pacific. During that campaign the L2 carrier phase tracking loop of the 4000 SD receivers had difficulty tracking signals at times of high ionospheric activity. The Ascension Island test took place between 11 and 16 November 1989. From an analysis of the collected data it was concluded that both the 4000 SST and TI 4100 receivers tracked through rapidly changing ionospheric conditions (average L2 phase changes of about 18 centimetres per minute) without significant L2 data loss.

Another test of GPS receiver performance was carried out in the auroral zone on a network near Yellowknife in the Northwest Territories of Canada [Langley et al., 1991a; 1991b]. Over a period of three days in June 1990, data was recorded at three sites with inter-site distances of about 50 and 100 km. At each site, three different commercially-available GPS receivers were used: an Ashtech LD-XII, a Trimble Navigation 4000 SST or STD, and a Wild-Magnavox WM102. Two observing sessions were held each day: one during which each receiver was fed by its own antenna, and one during which the three receivers at each site shared a common antenna through a signal splitter. Although the ionosphere was not all that active during the actual recording of data, it was nevertheless more active than normally encountered at mid-latitudes. One of the conclusions of the test was that the WM102 receiver performance seemed to be marginally better than that of the Ashtech and Trimble receivers, likely because of its use of P-code tracking on the L2 frequency. Leclerc [1991] has also examined the effect of ionospheric perturbations during a period of high solar activity. He concluded that such perturbations did not pose any serious impediments to the use of GPS in a kinematic mode for aerial photography applications.

Wanninger and Jahn [1991] have looked at the difficulties in processing GPS data collected in both the auroral and equatorial regions. They have found that the L_{43} (4 times the L1 observation minus 3 times the L2 observation) and L_{54} linear combinations are very helpful in some instances in resolving cycle slips and ambiguities in data taken under severe ionospheric conditions. Cocard and Geiger [1992] have also looked at the benefits of forming various linear combinations in the processing of GPS observations while Schaffrin [1992] has examined the proper choice of a linear combination which permits the calibration of the residual ionosphere over a particular area.

2.2 Predictions of Ionospheric Activity

Several agencies monitor the behaviour of the ionosphere and the earth's magnetic field and some provide predictions of future behaviour [e.g. Hruska and Coles, 1987]. These predictions may be useful in planning GPS observations so as to ensure that they take place during benign ionospheric conditions.

2.3 The Use of Satellite Positioning Systems for Studying the Ionosphere

Ionospheric scientists have used the Transit satellites as satellites of opportunity for studying the ionosphere for more than 30 years [e.g. de Mendonca, 1963; Leitinger et al., 1975;

1984]. By recording the Doppler shift on the two Transit frequencies, the change in TEC during a satellite pass may be determined. If data from a satellite pass can be acquired at several stations, it should be possible to obtain a two-dimensional image of ionospheric electron density by applying the techniques of computerized tomography [e.g. Austen et al., 1987]. The signals from the constellation of GPS satellites are also being used to study the ionosphere [e.g. Lanyi and Roth, 1988; Clynch et al., 1989; Melbourne, 1989; Coco, 1991]. Monaldo [1991] used dual frequency GPS data to assess spatial variability of the ionosphere and estimate its potential impact on the monitoring of mesoscale ocean circulation using data from altimetric satellites.

3. THE TROPOSPHERE

The troposphere is the lower part of the earth's atmosphere where temperature decreases with an increase in altitude. The thickness of the troposphere is not everywhere the same. It extends to a height of less than 9 km over the poles and in excess of 16 km over the equator [Lutgens and Tarbuck, 1989]. The presence of neutral atoms and molecules in the troposphere affects the propagation of electromagnetic signals. Atoms and molecules in the stratosphere also exist in sufficient numbers to affect the propagation of signals. However, since the bulk of the neutral atmosphere lies within the troposphere, the whole neutral atmosphere is often loosely referred to as the troposphere.

The range bias experienced by a signal propagating from a satellite to the ground may be expressed in first approximation by the integral equation

$$d_{trop} = \int_{r_s}^{r_a} [n(r) - 1] \sec y(r)\, dr + \left[\int_{r_s}^{r_a} \sec y(r)\, dr - \int_{r_s}^{r_a} \sec z(r)\, dr \right] \qquad (14)$$

where n is the refractive index, r is the geocentric radius, and y and z respectively denote the apparent and true satellite zenith angle. This equation holds for a spherically symmetric atmosphere for which n varies only as a function of geocentric radius. The first integral accounts for the difference in the electromagnetic and geometric lengths of the refracted transmission path. The bracketed integrals account for path curvature; i.e., the difference in the refracted and rectilinear path lengths.

The integral equation can be evaluated given knowledge of the actual refractive index profile or it may be approximated by an analytical function. In applications in satellite ranging, the latter approach is most common with the use of a closed-form or truncated-series approximation based upon a simplified atmospheric model. In most cases, water vapour and the dry gases are considered separately. Each component is written as the product of a zenith delay term, approximating the integral of the refractive index profile in the vertical direction, and a mapping function which maps the increase in delay with increasing zenith angle. In general form

$$\begin{aligned} d_{trop} = {} & D^z_{dry}(P_s)\ MF_{dry}(\psi_s, P_s, T(h)) \\ & + D^z_{wv}(T_s, RH_s, T(h))\ MF_{wv}(\psi_s, T_s, RH_s, T(h), RH(h)) \end{aligned} \qquad (15)$$

where

D^z_{dry} is the zenith delay due to dry gases
D^z_{wv} is the zenith delay due to water vapour
MF_{dry} is the dry gas mapping function
MF_{wv} is the water vapour mapping function

ψ_s	is the refracted zenith angle at the ground station
h	is the height above the ground station
T_s	is the surface temperature
P_s	is the surface pressure
RH_s	is the surface relative humidity
T(h)	is the temperature profile above the ground station
RH(h)	is the relative humidity profile above the ground station.

The refractivity of a parcel of air, $N = 10^6 (n - 1)$, is a function its temperature (T) and the partial pressures of the dry gases (P_d) and the water vapour (e):

$$N = K_1 \frac{P_d}{T} + K_2 \frac{e}{T} + K_3 \frac{e}{T^2} \tag{16}$$

where K_1, K_2, and K_3 are empirically determined coefficients. For radio frequencies up to about 30 GHz, the troposphere is non-dispersive (except for the anomalous dispersion of the water vapour and oxygen spectral lines) and hence N is independent of frequency. In radio meteorology, the equation for refractivity of air is most often written in the form

$$N = K_1 \frac{P}{T} + K_2' \frac{e}{T^2} . \tag{17}$$

where

$$K_2' = [(K_2 - K_1) T + K_3]. \tag{18}$$

The first and second terms of equation (17) are commonly referred to as the "dry" and "wet" components of refractivity. Alternatively, refractivity may be expressed as

$$N = K_1 \frac{M}{M_d} \frac{P}{T} - (K_1 \frac{M}{M_d} - K_2) \frac{e}{T} + K_3 \frac{e}{T^2} \tag{19}$$

where

$$\frac{M}{M_d} = \frac{T}{T'} = (1 + 0.3780 \frac{e}{P})^{-1} \tag{20}$$

in which T' is virtual temperature, and M and M_d denote the molar mass of moist and dry air respectively. The first term in equation (19) is sometime referred to as the "hydrostatic" component of refractivity [Davis,1986]. The formulation of equation (19) is useful as the zenith delay based on the hydrostatic component is not influenced by water vapour content unlike the "dry" component formalism.

Sea level values of the total tropospheric delay in the zenith direction are of the order of 2.3 to 2.6 metres, rising to in excess of 23 metres at zenith angles of 85°.

3.1 Profile Models and Mapping Functions

Over the past 20 years or so, geodesists and radio meteorologists have developed a variety of model profiles and mapping functions for the evaluation of the delay experienced by signals propagating through the troposphere at arbitrary zenith angles. The performance of some of these models has been assessed by Janes et al. [1989, 1991]. Delay predictions of the models and mapping functions were benchmarked against values obtained by ray-tracing the U.S. Standard Atmosphere and the associated Atmosphere Supplements which incorporate latitudinal and seasonal departures from the Standard. The authors concluded from their analysis that, of the models tested, the explicit form of the Saastamoinen zenith delay expressions [Saastamoinen, 1973] in combination with the Davis (CfA-2.2) hydrostatic [Davis, 1986] and Goad and Goodman water vapour mapping functions [Goad and Goodman, 1974] would provide superior performance to the other models under most conditions.

Mapping functions not tested by Janes et al. include one by Lanyi [1984] that incorporates both the hydrostatic and wet components, those by Ifadis [1986] who developed hydrostatic and wet mapping functions that can be site-optimized, and those by Herring [1992] which were based on the analysis of an extensive set of rawinsonde data from 11 sites in the United States. Herring reports that the typical r.m.s. difference between raytracing at a 5° elevation angle and his mapping functions is 30 mm for the hydrostatic delay and 10 mm for the wet delay. Davis et al. [1991] examined errors in the CfA-2.2 mapping function using data from a series of special VLBI experiments. They found that mapping function errors do not exhibit a coherent annual signature but rather appear to be random over the long term.

3.2 Water Vapour Radiometry

Although the hydrostatic component of the vertical delay can generally be well modelled using accurate surface values of barometric pressure, the same is not true for the wet component. The water vapour in the troposphere is not well mixed and its distribution is therefore usually spatially and temporally inhomogeneous. Water vapour radiometers (WVRs) have been developed in an effort to remotely sense the amount of water vapour along a ray path. A WVR measures atmospheric black body radiation which is affected by the presence of water vapour. Although the technology has evolved considerably [e.g., Elgered et al., 1991; Rocken et al, 1991], the use of WVRs, at least in VLBI, is reported to provide only a marginal improvement in accuracy [Kuehn et al., 1991a]. However, Tralli et al. [1988], based on the analysis of GPS data collected on baselines in the Gulf of California, suggests that the use of WVR data for tropospheric path delay calibration in humid regions appears to be important for achieving highest possible baseline accuracies. Experiments to further reduce and evaluate WVR instrumental errors are continuing [Kuehn et al., 1991b].

3.3 Estimation of Residual Biases

The residual tropospheric range bias remaining after the application of one of the zenith delay models and associated mapping function can, in most instances, be estimated using the range data itself. Such estimation can take the form of a single scale bias or residual zenith delay estimated for observations spanning many hours, hourly estimates, or stochastic estimation using the Kalman filter approach [Lindqwister et al., 1990]. Kalman filtering is an attractive alternative to water vapour radiometry both from the point of view of cost and accuracy. In fact, Tralli and Lichten [1990] have shown that stochastic estimation of total zenith path delays yields baseline repeatabilities of a few parts in 10^8, results which are comparable to or better than those obtained after path delay calibration using WVR and/or surface meteorological measurements.

3.4 Effect of Tropospheric Delay Errors on Small GPS Networks

Most tropospheric delay models and mapping functions for predicting tropospheric delay assume a laterally homogeneous atmosphere. In the actual atmosphere, the decorrelation of signal paths for a network of stations is governed by lateral gradients in atmospheric pressure, temperature, and humidity, and by differences in station elevation. Beutler et al. [1988] have shown that the effect of the differential troposphere on local GPS networks leads to a relative height error that can be written in a first approximation as

$$Dh_e = DD^z{}_t \sec y_{max} \tag{21}$$

where $DD^z{}_t$ denotes the difference in zenith delay between co-observing stations and y_{max} is the maximum zenith angle observed. Neglect of the differential troposphere leads to approximately 3 to 5 mm of relative height error for every millimetre change in zenith delay between stations for y_{max} = 70-80°.

Janes et al. [1991] intimate that users of GPS data processing software which incorporates a tropospheric delay model driven by surface measurements of temperature, pressure, and relative humidity should be aware of potential pitfalls when processing data collected on baselines of local scale. They suggest that modelling of the differential troposphere between co-observing stations is only advisable when the meteorological gradients clearly exceed the accuracy to which surface meterological parameters can be measured. Where horizontal gradients or station height differences are significant, careful measurement of surface meteorology is essential for proper modelling of the differential tropospheric delay. Temperature inversions, anomalous humidity profiles, and the use of inappropriate upper air profile lapse rate parameters can significantly reduce the accuracy of the delay model. Where gradients and height differences for a small network are slight, it may be more prudent to assume a laterally homogeneous atmosphere based either upon standard conditions (scaled to height) or upon averaged local meteorological measurements. Beutler et al. [1990] have also pointed out the difficulties associated with modelling the troposphere for small GPS networks.

3.5 Satellite Laser Ranging

In contrast to its properties at radio wavelengths, at optical wavelengths — from near ultraviolet, through visible, to near infrared — air is a dispersive medium. Hence at optical wavelengths, the refractivity of air is a function of wavelength. Another difference is that the refractivity of a parcel of air at optical wavelengths is only weakly dependent on the concentration of water vapour in the parcel and, in fact, the refractivity is actually reduced by the presence of the water vapour. For example, for a total pressure of 1000 mbar, a temperature of 15°C, and 0% relative humidity, the refractivity at a wavelength of 6328 Å is 272.90; increasing the relative humidity to 50% results in a refractivity of 272.53; and to 100%, 272.17 [Owens, 1967]. Contrast these values to the corresponding values for the radio refractivity of 269.30, 307.71, and 346.12.

The Marini and Murray [1973] model is commonly used in SLR for tropospheric path delay correction. But as has been pointed out by Janes et al. [1991] and others, this model has limited accuracy, especially at low elevation angles. Cohen and Pearlman [1989] report that efforts are underway to adapt some of the features of the CfA-2.2 mapping function to the optical part of the electromagnetic spectrum. However, tropospheric path delays at optical frequencies are less affected by water vapour than at microwave frequencies, so any improvement may be marginal. The accuracy of tropospheric delay corrections on SLR measurements has been considered by Bender [1992]. He has looked at the effect on model accuracy of horizontal gradients in atmospheric density as well as departures of the atmosphere from hydrostatic equilibrium. He suggests that it may be possible to correct for gradients using twice-daily gridded meteorological data or by estimating two gradient parameters from the laser data itself but that an even more attractive possibility would be the use of a dual frequency system as proposed by, among others, Abshire and Gardner [1985].

4. CONCLUSION

Although great progress has been made in understanding and modelling the effects of the atmosphere on satellite-based positioning systems in recent years, the atmosphere still remains a limiting source of error. If positioning systems are going to reach the one millimetre level (a goal set by the International Workshop on the Interdisciplinary Role of Space Geodesy [Mueller and Zerbini, 1989]), further effort will have to be expended in order to reduce the errors in ionospheric and tropospheric modelling. Current investigations into higher order ionospheric corrections, the distribution of water vapour in the troposphere, and the effect of horizontal gradients in atmospheric density, for example, all bode well for eventually reaching the one millimetre goal.

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